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THE STRUCTURE OF γ RAYS ON THE BASIS OF THE ELECTRO-MAGNETIC THEORY OF LIGHT.

By Jakob Kunz.

THE difficulties in the theories of radiation have become so great in the last few years that only a very fundamental new idea will be able to bring harmony into the present chaos of facts and theories. Before that solving idea appears, we can only draw all the logical conclusions which follow from a given hypothesis and compare these conclusions with experiments. This procedure leads to the result that neither one of the present hypotheses is able to coördinate satisfactorily all the facts. The electromagnetic undulatory theory of light explains the phenomena of reflection, refraction, interference, diffraction, etc., but it fails to explain the phenomena of radiation, the photoelectric and the related effects. The corpuscular theories and the quantum theories on the other hand are especially invented for the explanation of the latter group of phenomena, failing to account for the first group. Not only are the fundamental assumptions of the two theories different, but the older electromagnetic undulatory theory gives not only mathematical relations between the different quantities involved, but it visualizes at the same time, the phenomena, which are explained, so that we can see the mechanism of the processes. The recent quantum theories on the other hand give us only algebraic relations between different quantities such as the law of the photoelectric effect $\frac{1}{2}mv^2 = hn - V_0$, without giving us the least idea as to the mechanism of the phenomenon. It seems to be advantageous if not necessary for a theory to visualize the phenomena. Led by this idea, I shall show by the following figures and calculations that the electromagnetic theory of very hard Roentgen and γ rays leads to conclusions which remind us of an emission or corpuscular theory of rays. The problem has already been solved by A. Sommerfeld,[1] "Uber

[1] Sitzungsberichte der K. B. Akademie der Wissenschaften, 41, p. 1, 1911.

die Structur der γ Strahlen." I shall use, however, a different method which allows the visualizing of the remarkable conclusion of the electromagnetic theory.

An electron moves with the velocity v in the x direction; the electric force E at a distance r from the charge making an angle φ with the direction of motion is equal to

$$E = \frac{e\left(1 - \frac{v^2}{c^2}\right)\sqrt{x^2 + y^2 + z^2}}{\left[x^2 + (y^2 + z^2)\left(1 - \frac{v^2}{c^2}\right)\right]^{\frac{3}{2}}}.$$

x, y, z are the coördinates of the point in which the electric force E is measured, the zero of the system of the coördinates lies in the electron. We can express E also in the following way

$$E = \frac{e\left(1 - \frac{v^2}{c^2}\right)}{r^2\left(1 - \left(\frac{v^2}{c^2}\right)\sin^2\varphi\right)^{\frac{3}{2}}}.$$

The magnetic force H is given by

$$H = \frac{Ev \sin \varphi}{c}.$$

These equations tell us that with increasing velocity v the electromagnetic field following the electron, is more and more compressed as it were toward the equatorial plane, being perpendicular to the direction of motion.

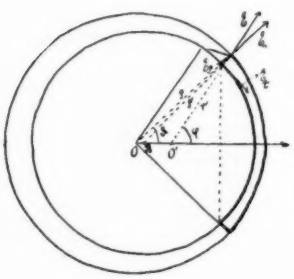

Fig. 1.

If now the electron comes into collision with a metallic plate it will come to rest during an interval of time t and move meanwhile over the distance l. If we draw the line of force t seconds after the collision began, we find the well-known disturbance within the Roentgen pulse, whose thickness δ is a function of ϑ given by the equation

$$\delta = \frac{l}{v}(2c - v \cos \vartheta).$$

Within the sphere with radius ρ' drawn round about B of Fig. 1 the final position of rest of the electron, we find the ordinary electrostatic field. Without the sphere with radius ρ, drawn about o, where the collision at

the moment $t = 0$ began, we find the electromagnetic field accompanying the electron in motion. The electric force E, pointing toward o′ where the electron at the moment t would be, if it had not been stopped, is equal to

$$E = \frac{e(1 - \beta^2)}{r^2(1 - \beta^2 \sin^2 \varphi)^{\frac{3}{2}}}; \quad \beta = \frac{v}{c}.$$

The component En of E in the direction ρ is equal to

$$E_n = \frac{e(1 - \beta^2) r \cos(\varphi - \vartheta)}{r^3(1 - \beta^2 \sin^2 \varphi)^{\frac{3}{2}}},$$

$$r \cos(\varphi - \vartheta) = \rho(1 - \beta \cos \vartheta),$$

$$r^3(1 - \beta^2 \sin^2 \varphi)^{\frac{3}{2}} = \rho^3(1 - \beta \cos \vartheta)^3,$$

$$E_n = \frac{e(1 - \beta^2)}{\rho^2(1 - \beta \cos \vartheta)^2}.$$

We shall now calculate the tangential electric force E_t in the shell. In Fig. 1 we consider the volume cut out of the shell by a cone with an angle ϑ. As there are no charges within this cap, the resultant flux of the electric force flowing out of the volume must be equal to zero. The inflowing flux ϕ_1 is due to the component E_t' and to the force e/ρ'^2 or approximately e/ρ^2

$$\phi_1 = E_t \delta 2\pi\rho \sin \vartheta + \frac{e}{\rho^2} 2\pi\rho(\rho - \rho \cos \vartheta),$$

$$= E_t \delta 2\pi\rho \sin \vartheta + e2\pi(1 - \cos \vartheta).$$

The outflowing flux ϕ_2 is due to the force E_n and is equal to

$$\phi_2 = \int_0^{\vartheta} E_n 2\pi\rho \sin \vartheta d\vartheta \cdot \rho$$

$$= e(1 - \beta^2) 2\pi \int_0^{\vartheta} \frac{\sin \vartheta d\vartheta}{(1 - \beta \cos \vartheta)^2}$$

$$= e(1 + \beta) 2\pi \frac{1 - \cos \vartheta}{1 - \beta \cos \vartheta}$$

$$\phi_1 = \phi_2,$$

hence

$$E_t \delta 2\pi\rho \sin \vartheta + e2\pi(1 - \cos \vartheta) = e(1 + \beta) 2\pi \frac{1 - \cos \vartheta}{1 - \beta \cos \vartheta}$$

$$E_t = \frac{e\beta \sin \vartheta}{\delta\rho(1 - \beta \cos \vartheta)} = \frac{ev \sin \vartheta}{\delta\rho(c - v \cos \vartheta)}.$$

In every electromagnetic disturbance the electrostatic energy is equal to the magnetic energy. This is the case if we assume for the magnetic

force H in the pulse the expression

$$H = cE_t.$$

The distribution of the lines of force is shown in Fig. 2, and in Fig. 3 for the limiting case where the velocity v becomes equal to the velocity

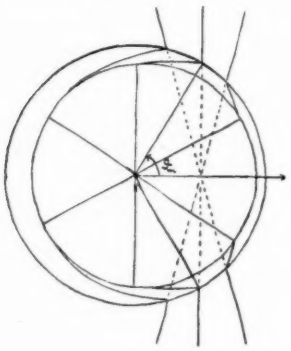

Fig. 2.

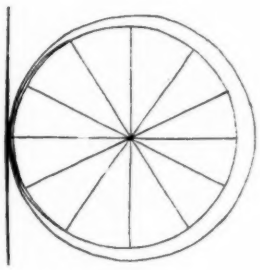

Fig. 3.

c of light. We see from these figures that with increasing velocity v the electric force E_t becomes a maximum in a certain direction ϑ_m where most of the energy flows away from the electron, and that in the limit for $v = c$ the energy flows out in the direction $\vartheta_m = 0$, that is in the direction of motion of the electron. When the velocity v is so small that it can be neglected with respect to c, then on the contrary, the maximum of the energy radiates away from the electron in the equatorial plane.

$$E_t = \frac{ev^2 \sin \vartheta}{l\rho(c - v\cos\vartheta)(2c - v\cos\vartheta)}.$$

If we neglect at first the change in the factor $1/(2c - v\cos\vartheta)$, we get

$$E_t = \frac{ev^2 \sin \vartheta}{l\rho(c - v\cos\vartheta)}.$$

and

$$\frac{\partial E_t}{\partial \vartheta} = \frac{e}{l\rho}\frac{v^2(c\ \mathrm{vos}\ \vartheta - v)}{(c - v\cos\vartheta)^2},$$

which vanishes when $\cos\vartheta = v/c$; for this angle the electric force is a maximum and the energy radiating away from the electron becomes approximately a maximum also.

When $\frac{v}{c} = 0.9$ then $\vartheta = 25^\circ\ 45'$,

when $\frac{v}{c} = 0.99$ then $\vartheta = 8^\circ$ and when

$\frac{v}{c} = 1$ then $\vartheta = 0^\circ$.

The effect becomes marked only when the velocity v approaches that of light. Returning to the complete expression E_t, we find that it becomes a maximum for the angle ϑ determined by

$$\cos^3 \vartheta - 2\left(1 + \frac{c^2}{v^2}\right)\cos \vartheta + 3\frac{c}{v} = 0.$$

For

$$\frac{c}{v} = \frac{10}{9},$$

the three roots are -2.42, 1.497 and 0.9210, hence the angle $\vartheta = 22° 55'$. But as the shell has not everywhere the same thickness, this angle does not accurately indicate the direction in which most of the energy is radiated away.

The energy of the electric force E_t within the elementary volume $2\pi\rho \sin \vartheta \rho d\vartheta \delta$ is equal to

$$dE_{es} = \frac{1}{8\pi} E_t^2 2\pi\rho \sin \vartheta \rho d\vartheta \delta$$

$$= \frac{1}{4} e^2 \frac{v^3 \sin^3 \vartheta d\vartheta}{l(2c - v\cos\vartheta)(c - v\cos\vartheta)^2}.$$

The electrostatic energy in the cap under the angle ϑ is given by the integral

$$E_{es} = -\frac{1}{4}\frac{e^2 v^3}{l}\int_0^{\vartheta} \frac{\sin^2 \vartheta d\cos\vartheta}{(2c - v\cos\vartheta)(c - v\cos\vartheta)^2}$$

$$= \frac{1}{4}\frac{e^2}{1}\left\{\frac{3c^2 - v^2}{c^2}\log\frac{c - v\cos\vartheta}{c - v} - \frac{4c^2 - v^2}{c^2}\log\frac{2c - v\cos\vartheta}{2c - v}\right.$$

$$\left. + \frac{c^2 - v^2}{c}\left[\frac{1}{c - v\cos\vartheta} - \frac{1}{c - v}\right]\right\}.$$

The function

$$f(\vartheta) = \frac{\sin^3 \vartheta}{(2c - v\cos\vartheta)(c - v\cos\vartheta)^2}$$

or

$$f(\vartheta) = \frac{\sin^3 \vartheta}{\left(2 - \frac{v}{c}\cos\vartheta\right)\left(1 - \frac{v}{c}\cos\vartheta\right)^2}$$

has been plotted in Fig. 4 for the values $v/c = 9/10$ and $v/c = 99/100$. This figure shows clearly that with increasing velocity the energy is radiated away in a direction which approaches more and more the direction of motion of the electron. The bearing of this conclusion on the fluctuations of Schweidler and perhaps on the difference in the

photoelectric effect according to the incidence of the beam of light, or Roentgen rays is obvious. If we attribute to the electromagnetic field not only energy, but also momentum and mass, then it follows, that the electromagnetic mass of the electron, when it comes to rest, is thrown

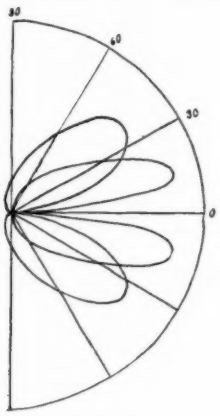

Fig. 4.

forward more and more with increasing velocity. This electromagnetic mass and momentum concentrated in a comparatively small space, is not so very different from the notion of light particles in the old emission theory.

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May 22, 1915.

ABSORPTION OF THE β-PARTICLES FROM SOME OF THE RADIOACTIVE SUBSTANCES BY AIR AND CARBON DIOXIDE.

BY ALOIS F. KOVARIK.

IT is a well-known fact that the absorption of the β-particles from some of the radioactive substances follows an exponential law, at least over a large part of the absorption curve when the ionization method of measurements is used. It was shown by Kovarik and McKeehan[1] by a statistical method that the numbers of β-particles are not diminished exponentially when the β-particles pass through matter, this, however, does not decrease the value of the ionization method because of its simplicity in application in checking up β-radiations of any radioactive material. In the ionization experiments, the absorbing material used is generally solid material, and usually, aluminium or tin foils. The solid foils can never be obtained uniform in thickness, if thin foils are desired, as, for example, in studying soft β-radiations. It would, therefore, seem desirable to devise a method of investigation of the absorption of the β-particles, by the ionization method, in which the absorbing material would be a gas which would permit uniform layers of absorbing molecules and also give possibilities of use in the cases where layers of relatively few absorbing molecules might be desired. This was done and the exponential law was found verified for gases for the rays for which the absorption curve was exponential for aluminium. A determination of the absorption coefficient was then deemed also desirable; likewise an investigation of the relation of activities of the soft and hard radiations in case of some of the radioactive substances.

The first determination of an absorption coefficient in a gas is due to Eve[2] who obtained a value for the coefficient of absorption of the heterogeneous radium *C* rays in air. His method consisted in measuring the ionization produced in a β-ray electroscope when the source of radiation was placed at different distances from the electroscope and applying the inverse square law. This method is readily applicable to strong and penetrating radiation where the distances may be so great that the size

[1] Kovarik, Alois F., and McKeehan, L. W., Phys. ZS., 15, p. 434, 1914.

[2] Eve, A. S., Phil. Mag., S. 6, 22, p. 8, 1911.

of the electroscope and the general disposition will affect the results in only a slight manner. He obtained $\mu = .0045$ cm.$^{-1}$ air. Shortly after that, the writer[1] published abstracts of the preliminary work. Since then, a paper on the absorption of the β-particles of uranium in air by Florance[2] has appeared.

In a former investigation[3] of the β-radiations by the writer, it was first pointed out that the soft radiation of Radium *D*, *E*, *F* is most likely due to radium *D*. This was soon confirmed by v. Beyer, Hahn, and Meitner[4] by a photographic method applied to the magnetic spectra. They showed in fact that radium *D* has two line spectra, one corresponding to rays having an absorption coefficient of about 300 cm.$^{-1}$ aluminium and the other very much softer. In the present investigation the method used made it possible to separate these two radiations by studying their absorption curves in air.

METHOD AND APPARATUS.

The method employed is, in fact, the same in principle as that used by Bragg and Kleeman in their study of the ionization curves of the α-particles. An ionization chamber 2 mm. deep (*AB*, Fig. 1) consisting of a circular brass plate 7.4 cm. in diameter, connected electrically by a pair of quadrants of an electrometer, and a thin aluminium foil, 0.003 cm. thick, connected to one end of a high potential battery, whose other end was to earth, was fixed in position. A guard ring in the same plane as the circular plate of the ionization chamber and held rigidly to it by means of amber bridges with a space of 1 mm., helped to form a uniform field in the ionization chamber. The source of β-rays was placed at *D* on a central axis perpendicular to *A* and at any distance not greater than 10 cm. from the ionization chamber. Between the source and the high

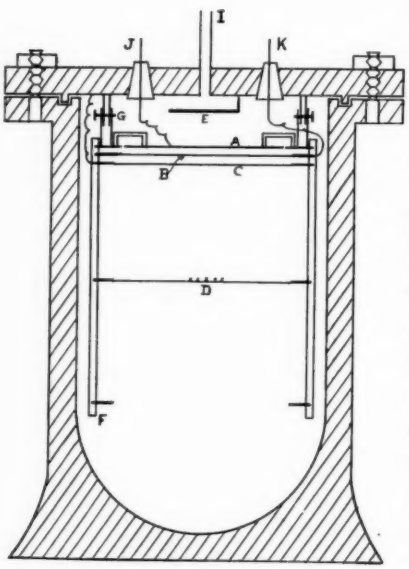

Fig. 1.

[1] Kovarik, Alois F., PHYS. REV., XXXIV., p. 142, 1912; PHYS. REV., N. S., 3, p. 150, 1914.
[2] Florance, D. C. H., Phil. Mag., S. 6, 25, p. 172, 1913.
[3] Kovarik, Alois F., Phil. Mag., S. 6, 20, p. 849, 1910.
[4] v. Beyer, O., Hahn, u. Meitner, Phys. ZS., 12, p. 378, 1911.

potential plate B was another foil (.0003 cm. aluminium) connected to earth, forming an ion trap preventing disturbances should the foils of aluminium have small holes. The foils of B and C were attached to rigid iron rings which along with the guard ring of A were held in position by being set into slits in 3 ebonite rods, F. The whole was attached by means of screw connectors, G, to the removable cover of the apparatus cylinder for the gas. This cylinder was a strong brass holder 18 cm. deep, and 10 cm. in diameter. The gas entered through I from a tank containing the gas at high pressure. A gas manometer consisting of air enclosed in a long (130 cm.) capillary glass tube by a thread of mercury 5 cm. long was in communication. The capillary tube was carefully calibrated and the pressures were deduced by Boyle's law. E was a metal shield protecting the ionization chamber from the stream of gas from I. The air used was dry, whereas the carbon dioxide was the ordinary commercial product and quite moist. The quadrant electrometer used was of the Erikson-Dolezalek type.[1] On account of the weak material available in some cases and on account of the small volume of the ionization chamber a sensitive instrument was necessary. The Erikson method of charging the needle makes it a simple matter to use the finest quartz fibers available without altering their coefficient of torsion. The sensibility of the instrument used was 2,500 mm. per volt, the scale being 150 cm. from the mirror of the instrument.

In some of the experiments the rate method of observation was employed but on account of varying electrical leaks depending on potential of the charge conveyed to the quadrants, the null method of Townsend[2] was substituted and found far more satisfactory.

The β-particles coming from D radiate in all directions and some pass within the frustum of the cone defined by the plane circular source used at D and the ionization chamber. The particles are subject to absorption by the gas between B and D and also by the gas within the ionization chamber itself. If the pressure of the gas in the vessel is increased, the absorbing medium is proportionately increased in quantity but the ionization is also increased. The increase in ionization is, within the pressures used (20 atmospheres), proportional to the pressure. Consequently, the observed ionization currents at the various pressures were reduced to what they would be at one atmosphere. By plotting these values against pressure in atmospheres, one obtains an absorption curve and a coefficient of absorption per atmosphere of the gas used can be deduced for any particular disposition of the source D. To compare

[1] Erikson, H. A., PHYS. REV., N. S. 1, p. 253, 1913.

[2] Townsend, J. S., Phil. Mag., S. 6, 6, p. 598, 1903.

results, it is necessary to know the depth of the absorbing layer of gas, and since the β-rays are not parallel but radiate in all directions, it is necessary to obtain the mean distance traversed by a β-particle through the gas. Disregarding deflections in its path this distance would constitute the mean path of the β-particles. This distance was obtained graphically very readily, the ionization chamber being circular, the source being also circular and symmetrically placed below the chamber. Since the gas in the ionization chamber also absorbs the radiation, the measurements were made to a plane midway between the plates of the ionization chamber. The coefficient of absorption could then be deduced in $cm.^{-1}$ or $(gm.\ per\ cm.^2)^{-1}$ of the gas used.

It is to be noted that the potential difference between A and B must be sufficient for a saturation current. For the distance of 2 mm. between A and B, 2,000 volts is sufficient for the pressures used.

Since the β-particles are readily deflected in their course, and considerab'y reflected by brass, some rays might enter the ionization chamber by a roundabout way. To prevent such reflected rays having effect in this manner a cylinder of paper, which reflects[1] only a small percentage of the incident radiation, was wrapped around the ebonite rod F before inserting the apparatus into the vessel. The effect of the incident radiation then became negligible.

With the entrance of a gas into the vessel a charging up of the amber insulation plugs took place and time had to be allowed before consistent readings could be taken. This was, of course, a source of annoyance especially in the case of rapidly disintegrating substances.

The absorbing value of the thin aluminium foil B and C had to be taken into consideration in the case of soft rays. The natural leak of the apparatus at different pressures was obtained and readings with the rays were properly corrected.

RESULTS.

It was first shown that the radiations which are absorbed exponentially by aluminium are also absorbed exponentially by gases. The curves of Fig. 2 show the result for the rays of radium E absorbed by air and carbon dioxide, the mean path being 10.4 cm. These logarithmic curves show perfect resemblance to similar curves with aluminium, which are well known. The same thing was found with the other radiations investigated, all giving a straight line for that part of the logarithmic curve which was found to obey the exponential law with aluminium absorbing foils.

In order to see whether the mean path obtained graphically is correct,

[1] Loc. cit.

observations were made on the absorption coefficient of the β-rays from radium *E* when the source was placed at different distances from the

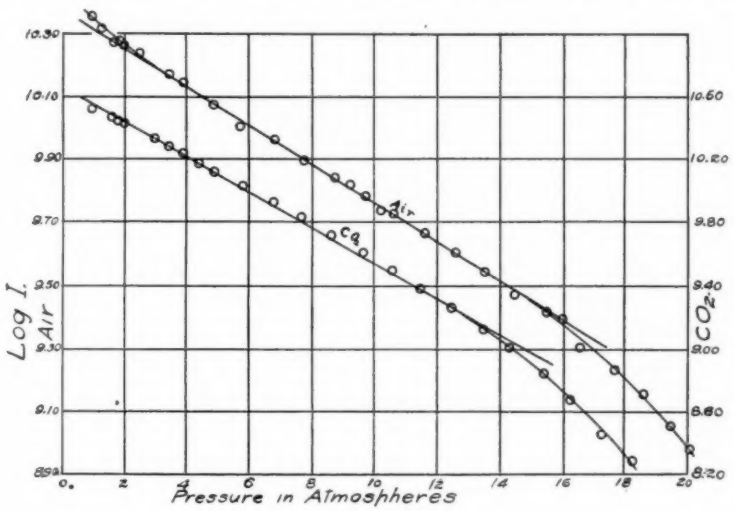

Fig. 2.

ionization chamber. The results are given in Table I., where the coefficients of absorption expressed per cm. of air are entered against the

TABLE I.

Radium D, E, F at Different Distances from the Ionization Chamber.

Mean Path of the β-Particles in Cm. of Air.	Absorption Coefficient in Cm.$^{-1}$ Air at 1 Atmosphere and 22° C.
10.60	0.0151
5.36	.0145
3.00	.0149
2.31	.0150
1.64	.0160

mean path obtained graphically. The agreement is good except for the shortest path in which case the rays varied much in the length of path.

The radioactive substances used were radium *D*, *E*, *F*, one specimen chemically separated and another prepared from radium emanation. These gave the radiations from radium *E* and also from radium *D*. The actinium active deposit on thin aluminium was used for actinium *B* and *D*, and thorium active deposit for thorium *B* and *C* + *D*. Uranium *X* chemically separated and also uranium oxide were used for the soft and hard rays of UX_1 and *UX*. The soft radiations were separated from the hard radiations by subtracting the latter from the observed results, calculating their values for low pressures according to the exponential law. There was no difficulty in doing this when observa-

tions were carefully carried through with small pressures and when the mean path was short. The results are given in Table II., in which the values of the coefficients are expressed for air and carbon dioxide both in cm.$^{-1}$ at one atmosphere and 22° C. and also in (gm./cm.^{2})$^{-1}$.

TABLE II.

Coefficient of Absorption.

Source of β-Particles.	Cm.$^{-1}$ Air 1 Atm. and 22° C.	Air (Gm./Cm.^{2})$^{-1}$.	Cm.$^{-1}$ CO_2 1 Atm. and 22° C.	CO_2 (Gm./Cm.^{2})$^{-1}$.
Radium E	.0152	12.70	.0297	16.31
Actinium C+D	.0091	7.60	.0175	9.62
Thorium C+D	.0068	5.68	.0129	7.08
Uranium X_2	.0065	5.43	.0114	6.26
Uranium X_1	.12	100.	.23	126.
Radium D	.097	81.	.183	101.
Radium D (very soft)	.64	535.	1.69	930.
Thorium B	.090	75.	.142	78.
Actinium B	.31	260.		

In order to obtain an idea of the relative ionizations produced by soft and hard rays when both are present, extrapolation exponentially to zero thickness of the absorber was carried out, and the ratio of the ionization produced by the soft rays to that produced by the hard rays in air at one atmosphere was averaged for the particular radiations in the various experiments. These values are given in Table III. It will be noted that

TABLE III.

Relation of Ionizations in One Atmosphere of Air of Soft Radiations to Hard Radiations Reduced by Extrapolation to Zero Absorber.

Sources of β-Particles.	Ratio of Ionizations.
$\frac{\text{Radium D (more penetrating)}}{\text{Radium E}}$	0.30
$\frac{\text{Radium D (very soft)}}{\text{Radium E}}$	2.4(?)
$\frac{\text{Actinium active deposit (soft)}}{\text{Actinium active deposit (hard)}}$	1.14
$\frac{\text{Thorium active deposit (soft)}}{\text{Thorium active deposit (hard)}}$	2.86

the value for the soft thorium radiation compared with the hard is higher than that usually given. The value for the very soft radium *D* radiation must be taken with reserve, because the extrapolation was too great, but that for the more penetrating radiation of radium *D* seems satisfactory; the same is true of the relation of the soft to the hard radiation of actinium active deposit.

CONCLUSIONS.

1. A method depending on the change of pressure of a gas was devised for measuring the absorption of the β-particles. This consists of measuring the ionization in a shallow ionization chamber placed at a fixed distance from a source of radiation within a high pressure vessel: reducing the ionization readings at various pressures to that at one atmosphere of pressure, measuring graphically the mean path of the β-particles, and then, in the usual manner, deducing the coefficient of absorption.

2. It was shown that the absorption curves in gases are the same in shape as those obtained with aluminium foils.

3. It was shown that the graphical method of obtaining the mean path is correct.

4. Values of the absorption coefficients in air and carbon dioxide of the β-particles of some of the radioactive substances were obtained.

5. Relation between the ionizations of the soft and the hard radiations were computed by extrapolation to zero absorber. This was done for the two soft radiations of radium *D* compared with the effect of the radiations of radium *E*; soft and hard rays of the actinium active deposit; soft and hard rays of the thorium active deposit; soft and hard rays of uranium *X*.

In conclusion I desire to express my deep gratitude to the committee of the Minnesota Research Fund for allowing funds to procure some of the necessary apparatus and material to carry out this work.

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THE UNIVERSITY OF MINNESOTA,
June 11, 1915.

DISTRIBUTION OF TRANSMITTED AND REFLECTED β-PARTICLES DETERMINED BY THE STATISTICAL METHOD.

BY ALOIS F. KOVARIK AND L. W. MCKEEHAN.

THE scattering of cathode and β-particles has been studied by a number of investigators. Villard[1] observed the separation of β from γ rays by the scattering of the former, using a photographic method. Crowther[2] obtained "complete" scattering of β-rays from uranium by aluminium 0.015 cm. thick or gold 0.0002 cm. thick. This refers to the thickness beyond which the absorption of an originally parallel beam of rays became exponential. Kovarik[3] showed the importance of scattering in measurements of absorption with different experimental dispositions. Wilson's[4] photographs of the tracks of cathode and β-particles show the scattering of individual particles. McKeehan's[5] work on the scattering of cathode rays in air showed the presence of single scattering in the case of rays much slower than β-rays. Scattering was proposed by us[6] to explain certain initial peculiarities in curves for combined absorption and reflection of β-rays.

The theory of single scattering of an electrified particle given by Rutherford[7] and experimentally verified by Geiger[8] in the case of α-particles, seems the most probable basis for any explanation of the scattering of β-rays, including under this term all changes from the original direction of projection.

Measurement of the number of β-particles distributed in any direction by a plate of metal placed in the primary beam, should afford a test for any such explanation. By the statistical (or counting) method here employed, such numbers are obtained directly, and should be more conclusive than numbers deduced by interpreting the results of ionization experiments.

[1] Villard, P., Ions, Electrons, Corpuscles, p. 1024, 1905.
[2] Crowther, J. A., Proc. Roy. Soc., A 80, p. 186, 1908.
[3] Kovarik, Alois F., Phil. Mag., S 6, 20, p. 849, 1910.
[4] Wilson, C. T. R., Proc. Roy. Soc., A 87, p. 277, 1912.
[5] McKeehan, L. W., PHYS. REV., N. S. 4, p. 140, 1914.
[6] Kovarik, Alois F., and McKeehan, L. W., Phys. ZS., 15, p. 434, 1914.
[7] Rutherford, E., Phil. Mag., S 6, 21, p. 669, 1911.
[8] Geiger, H., Proc. Manch. Lit. and Phil. Soc., 55, Pt. II., p. xx, 1911.

Apparatus and Manipulations.

In our previous paper[1] we described the general method here employed. Certain alterations adapting the method to the experiments in hand were found necessary. In order to obtain the distribution of the rays in azimuth, a source of β-rays and a square radiating plate 6 cm. on the edge could be fixed in any desired positions relative to each other, and then could be rotated as a whole about an axis lying in the radiating plate. Two cylindrical counting chambers were fixed in position facing each other, and the axis of rotation bisected their common axis so that simultaneous readings could be taken of the numbers emitted from the radiating plate on the side of transmission and on the side of reflection. The apparatus was made especially rigid and accurate in its working parts. Each counting chamber consisted of a brass tube of inside diameter 1.3 cm. and 7.5 cm. long. Two diaphragms each having a central opening 0.165 cm. in diameter were fixed 1 cm. apart in this tube making the depth of the counting chamber 2.3 cm. The remainder of the tube in front of the first diaphragm formed a shield to exclude extraneous rays and was beveled on one side so that the inner surface would not be exposed to direct radiation from the source with consequent large reflection. The chambers with their points were interchangeable. All metal surfaces exposed to the rays, and from which it might be possible for particles to reach either counting chamber by reflection, were covered with cardboard 0.3 cm. thick.

Discharging points of steel and of platinum were used. In the case of platinum the most satisfactory points were obtained by momentarily heating in a flame the tip of a short piece of platinum wire 0.005 cm. in diameter, set in a brass rod. The platinum fused into a beautifully smooth knob, the size depending upon the heat applied and its duration, and by suitable manipulation a much smaller knob could be produced at the extremity of the first. Such a point, if symmetrical, gave excellent results, but continued use caused disintegration of the surfaces and the destruction of the point. Contrary to the experience of Chadwick[2] a more permanent point could be obtained by grinding a hard steel sewing needle to an exceedingly tapering cone which was then heated to redness in an alcohol flame. The resulting surface was smooth and less affected by use than that of the platinum points.

To avoid the errors due to gradual change in the conditions of discharge from the points they were placed at frequent intervals in a standardizing counting chamber of fixed dimensions and distance from a source of α-

[1] Loc. cit.

[2] Chadwick, J., Verh. d. D. Phys. Ges., 16, p. 383, 1914.

or β-rays. The standard sources used were, for the α-rays, a polonium plate, and for the β-rays, a radium *D*, *E*, *F* plate. The number of α-particles emitted by the polonium plate was checked by the electrical and scintillation methods. The number to be expected by the statistical method was then calculated, and points giving values in good agreement with this were considered to be in good working condition if no regularity in sequence or grouping of the deflections in the electrometer (*e. g.*, double deflections) could be noted, and if the individual deflections were uniform in size and fairly large. Two standards were used because the first effect of deterioration was noticeable with β-rays. Faulty points give high values firstly, because discharges occur without the presence of rays and secondly, because a single particle causes a series of two or three discharges in quick succession. The β-ray standard was fixed to give about 30 particles per minute in the counting chamber and at this rate the occurrence of double deflections should be rare; hence the occurrence of many "doubles" in this standardization gave good grounds for rejection of the point.

Points which satisfy these tests do so over a range of applied potential difference of more than 50 volts in a total of 1,500 volts, the exact range and maximum depending upon the point. With all these precautions the method is highly satisfactory, contrary to the conclusions of certain other investigators.[1]

Two string electrometers of the Lutz-Edelmann type, one connected to

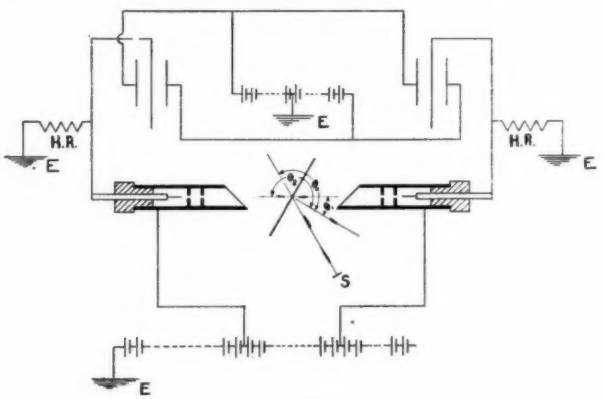

Fig. 1.

each discharging point, were used for visual observation, the sensibilities being adjusted to be approximately equivalent. The electrometer fibers were 0.0006 cm. in diameter. The general disposition of electrical con-

[1] Myssowsky, L., and Nesturch, K., Ann. d. Phys., 43, p. 461, 1914.

[2] Loc. cit.

nections was the same as in our previous work[2] so modified as to permit the use of different potentials on the two counting chambers when the points required this difference. The apparatus and connections are shown in diagram in Fig. 1.

RESULTS.

The number of particles entering the counting chamber per minute was obtained for each of the values of θ shown in Fig. 2, in which the normal to the radiating plate on the same side as the source is taken as the line of reference. It is to be noted that in reality this plate and the source were rotated while the counting chambers remained at rest. A series of counts with frequent interchange of points, counting chambers, and angles, was taken until the aggregate time of counting for each position was long.

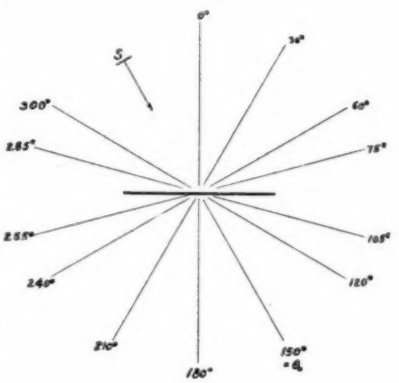

Fig. 2.

The weakness of the available source, a radium *D*, *E*, *F* plate, made it necessary to use small dimensions and a non-parallel beam of rays incident on the radiating plate. The difficulty of calculating the numbers to be expected on certain assumptions regarding the distribution on emergence, to be used in comparison with the experimental results, is unfortunately much increased by this fact, but the corrections for air scattering and absorption become relatively less important. Readings at or near a grazing emergence were omitted because the plate in these positions does

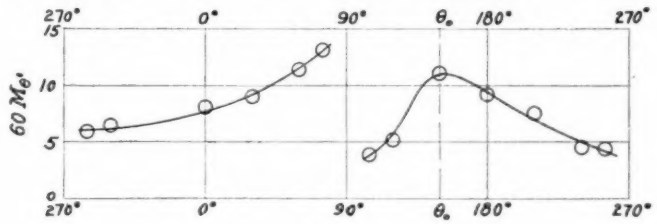

Fig. 3.

not cover the base of the cone defined by the diaphragms of the counting chambers. The thickness of the radiating plate (.0856 gm. Al per cm.^{2}) was selected for two reasons: firstly, this thickness makes the numbers of β-particles emitted from the two sides approximately equal; secondly, this thickness is much greater than the thickness given by various investigators as that necessary for "complete" scattering. The results

obtained in the experiment are shown in Fig. 3. Angles between 270° and 90° correspond to reflection, angles between 90° and 270° to transmission, according to the usual meaning of these terms.

Similar curves for reflection only were obtained with a less rigid form of apparatus using thick and thin plates of tin and a thick plate of carbon, respectively, the dissymetry in the curves being more marked for the thinner or lighter plates.

Curves for transmission through aluminium plates of different thicknesses are shown in Fig. 4, the same variables being plotted as in Fig. 3,

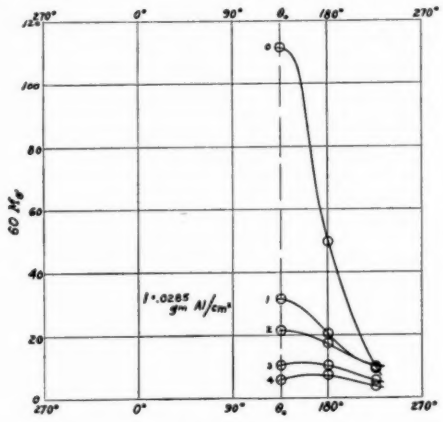

Fig. 4.

but the positions of the source and counting chamber being different as indicated. The suppression of the maximum in the original direction of the beam of rays is very noticeable.

Discussion and Explanation.

The path in air between large deflections of β-particles, as suggested by Wilson's photographs[1] of their tracks, agrees in order of magnitude with the air equivalent of the thinnest aluminium foil (.0008 gm. Al per cm.^{2}) used in our previous work, which gave a marked scattering effect.

This suggests single scattering as the cause of the effect noted, meaning by single scattering the deviation in direction of a β-particle due to a single charged center. We shall assume such single scattering to be the chief cause of the distributions observed above. The gradual curvature of path, perhaps due to the simultaneous action of many central forces, and also shown in the photographs above mentioned, is not excluded by this hypothesis, but is regarded as of secondary importance in determining the final distribution in direction.

[1] Wilson, loc. cit.

Assume that β-particles of velocity v_0 are incident upon a plate of matter of thickness t and of indefinite extent. We wish to know the probable distribution of the emergent β-particles in any plane perpendicular to the plate.

Let $n_0 d\omega$ be the number of particles incident per second upon a unit area of the plate in directions lying within a small cone of solid angle $d\omega$ the axis of which, produced through the plate makes an angle θ with the normal to the plate on the side of incidence. The total number of particles incident per second upon unit area, or

$$N_0 = \int n_0 d\omega.$$

Similarly, let $n_1 d\omega$ be the number of particles moving in directions lying within $d\omega$ after their first deflection, at whatever depth this occurs. The total number of particles deflected at least once of those incident per second upon unit area, is

$$N_1 = \int n_1 \, d\omega.$$

Extend this notation up to m deflections. The distribution n_m is a function of n_0, v_0, and t. Also $N_{m+1} < N_m$ since some particles included in N_m escape from the plate without suffering a further deflection, and some are stopped completely. Denote the number escaping by N_m', their distribution with angle by n'_m, and let n be the distribution of all the rays escaping from the plate, then

$$n = \sum_{m=0}^{m=\infty} n_m'.$$

Now let $n_m' = a_m n_m$ in which the factor a_m, which must be less than unity, depends upon the following variables:

1. The distribution within the thickness of the plate of the location of the mth deflection; this in turn is a function of m, n_0, v_0, and t

2. The velocity v_m of the rays after the mth deflection; this in turn is a function of m, v_0, and also of θ when m is small

3. The value of θ. The thickness of material traversed after the mth deflection is a function of θ and is least when $\theta = 0°$ or $\theta = 180°$.

The effect of the first variable may be estimated in a few cases. Let the mean depth of the mth deflection measured from the incident side be $\bar{d}_m$. The following relations are obvious.

$$\lim_{t=0} \bar{d}_1 = \tfrac{1}{2}t,$$

$$d_m > d_{m-1},$$

and

$$\lim_{m=\infty} \frac{\bar{d}_m}{\bar{d}_{m-1}} = 1,$$

for all values of t.

The value of a_m cannot depend upon $\bar{d}_m$ alone but rather upon the distribution of the mth deflections, as a function of the depth, say d_m, but in general an increase in $\bar{d}_m$ will increase a_m on the side of transmission and decrease a_m on the side of reflection.

The effect of the second variable, v_m, is to make a_m decrease as m increases because velocity is reduced by deflections.

The effect of the third variable, θ, is of the same nature as the effect upon the intensity of luminous radiation received from an incandescent plate viewed obliquely, and is due to the absorption of a volume radiation in the material of the radiator. If the plate is thick enough to deflect a number of times practically all the rays that fall upon it, the law of emergence as far as this variable is concerned is identical with that for luminous radiation from a solid and we may put

$$a_m = b_m \cos \theta,$$

in which b_m depends on θ only if n_0 or v_m do so, thus affecting the first two variables discussed above.

The volume radiation here described is in sharp contrast to the surface radiation and consequent uniform distribution due to a thin film of radioactive material, such as radium active deposit.[1]

Substituting the value of a_m in the expression for n we may write

$$n = \cos \theta \sum_{m=0}^{m=\infty} b_m N_m \left(\frac{n_m}{N_m}\right),$$

wherein n_m/N_m depends only upon m, n_0/N_0, and upon the law of scattering while b_m and N_m depend also upon t, and in the general case the former may depend upon θ. In the apparatus actually employed, however, θ_0 varied between very narrow limits at any point of the plate, since the source was small in area, and we may then safely assume that $b_m N_m$ is here independent of θ though not independent of θ_0.

Since a similar sum will express the emergent rays for any value of v_0 we may assume the coefficients to correspond to any initial distribution of velocities. The emergent rays are necessarily heterogeneous in any case.

The number of particles entering the counting chamber per second when it is placed at the angle θ', measured at the center of the plate, is the surface integral

$$M\theta' = \int n\Omega d\sigma = \int N_0 \left(\frac{n}{N_0}\right) \Omega d\sigma,$$

in which Ω is the small solid angle at the surface element $d\sigma$ within which

[1] Rutherford, E., Phil. Mag., S6, 12, p. 152, 1906.

the path of an emergent particle must lie in order that it may enter the counting chamber. The angle Ω and the inclination of its axis to the normal, θ, which determines the appropriate value of n, depend only upon θ', the position of $d\sigma$, and upon the dimensions of the apparatus.

Preliminary experiments showed that N_0 varied inversely as the square of the distance from the source to $d\sigma$ except for the absorption of the intervening air and directly as $\cos \theta_0$, within the region to be used.

Values of $M_{\theta'}$ were calculated for the experimental dispositions of source, plate, and counting chamber, and for the following different assumptions regarding the distribution of emergent particles

$$\frac{n}{N_0} = k, \frac{n}{N_0} = k' \cos \theta, \quad \frac{n}{N_0} = k'' \cos^2 \theta.$$

More general distribution laws were not chosen because of the fact that for each of those tested the final results were as well expressed by the simple relation $M_{\theta'} = n/\cos \theta'$, where n corresponds to the value of θ',

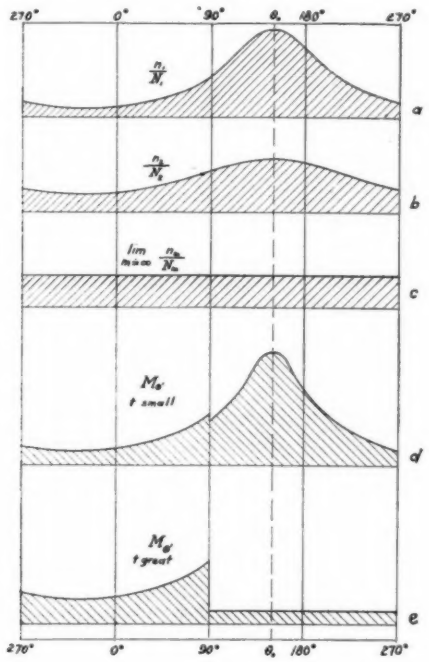

Fig. 5.

as by the integral actually used in the extended computations. Calculation also shows that the principal contributions to $M_{\theta'}$ are from a small area near the center of the plate.

Upon the results of these computations we therefore base the following equation as a first approximation

$$M_{\theta'} = \sum_{m=0}^{m=\infty} b_m N_m \left(\frac{n_m}{N_m}\right).$$

The approximation is still closer if the distribution is symmetrical about the normal to the plate.

Experiments to be quoted below make it probable that the mean angle of deviation at a single deflection is fairly large. Assuming this we may predict the approximate shape of the curves connecting n_1/N_1, n_2/N_2, $\lim_{m=\infty} n_m/N_m$, and θ if the incident beam consists of parallel rays.

Such distribution curves are sketched in Fig. 5 (*a*, *b*, and *c*). The maximum remains at θ_0 but becomes less marked as m increases.

Consider the variations in value of the coefficients $b_m N_m$ as t increases from 0 to ∞, and the effect upon $M_{\theta'}$ of these variations. For small values of t the values of N_m form a rapidly diminishing and convergent series. The values of b_m are nearly equal to unity for small values of m; they decrease with increasing m on account of the corresponding decrease in velocity; on the transmission side they increase slightly with increasing m and $\overline{d}_m$, and on the reflection side they decrease slightly with increasing m and $\overline{d}_m$. On the whole we may put

$$M_{\theta'} = \overline{N}_0 \left\{ (1 - c)\frac{n_0}{N_0} + c\frac{n_1}{N_1} \right\},$$

in which c is small compared to unity. Such a distribution is shown by Fig. 5 *d*. For large values of t the values of N_m diminish less rapidly than above, but the series is still convergent. On the reflection side the values of b_m decrease with increasing m as above, but more rapidly, since $\overline{d}_m$ increases to a greater limiting value than in that case. More terms must be taken before it can be assumed that the sum of the remaining terms is negligible, and since each term so taken follows more nearly a uniform distribution the only change from the previous case is seen in a raising of the whole curve without much change in its shape. On the transmission side the values of b_m increase as m increases (until the increase in $\overline{d}_m$ is overcome by the decrease of v_m), and at first more rapidly than N_m decreases, so that terms for either low or high values of m are less important than some of the intermediate terms. This justifies the equation

$$\lim_{t=\infty} M_{\theta'} = c'\overline{N}_0,$$

where

$$\lim_{t=\infty} c' = 0.$$

expressing a uniform and finally vanishing distribution on the transmission side. Such distributions are shown by Fig. 5 *e*. The experimental curve for $M_{\theta'}$ (Fig. 3) is seen to be intermediate in character between the curves of Fig. 5 *d* and *e*.

If the distribution, n_0, is similar to that at a point near a thin plane source of radioactive material, b_m will depend on θ and a more general reasoning than that given above must be employed. Let the plate be so thin that the probability of even a single deflection is very small for rays incident normally. For rays incident obliquely, however, the probability of a deflection is greater and some of the deflected rays emerge normally. This means that the number emergent normally from such a very thin foil is greater than the number incident normally. The thickness producing this effect is of the same order of magnitude as the mean free path of the β-particles in the material, and the observed rise will be the more marked the greater is the average deviation at a single deflection. For somewhat greater thickness the original uniform distribution will nearly reëstablish itself, but this will be replaced at still greater thickness by the cosine distribution due to volume emission. If the stoppage is small, as in aluminium, this may correspond to a second maximum in $M_{180°}$ reached more gradually than the first.

Exactly these effects are seen in the case of combined absorption and reflection of β-particles in our previous work,[1] when the absorbing (or scattering) foils were placed in the incident beam close to the source. The sharp initial rise, subsequent drop, and further gradual rise are due to the variations in the number of rays received near the center of the reflector (and therefore near the normal to the absorbing foils). The thickness of aluminium causing the maximum initial rise was 0.0006 cm., so that the "free path" of these β-particles in aluminium is of this order of magnitude. We deduced about the same value from Wilson's photographs of the β-particle tracks in air, by application of the density relation. In the lower curves of the figure referred to above, the second maximum does not appear for the absorbing foils in the reflected beam, because an approximate cosine distribution has already been produced by the reflection. The initial rise is also much reduced for the same reason.

Conclusions.

1. The statistical method of Geiger has proven satisfactory for accurate work on β-radiations with proper methods of experimentation and checks upon the results.

2. This method has been applied to the simultaneous determination of

[1] Kovarik and McKeehan, loc. cit., see Fig. 6.

the distribution of the particles from radium *D*, *E*, *F* transmitted and reflected by a plate of aluminium of medium thickness.

3. These results and those given in a previous paper have been interpreted in accordance with Rutherford's theory of single scattering.

4. The rays from a thick radiating plate emerge on the transmission side in a manner similar to the emission of light from a heated solid, *i. e.*, according to a cosine law.

5. From the phenomenon of initial scattering described in a former paper we have deduced that the mean path for these β-particles between deflections in aluminium is of the order of 10^{-3} cm.

In conclusion we desire to express our appreciation of the financial assistance afforded us under the provisions of the Minnesota Research Fund.

PHYSICAL LABORATORY,
THE UNIVERSITY OF MINNESOTA,
June 11, 1915.

A METHOD OF MEASURING HEAT CONDUCTIVITIES.[1]

BY R. W. KING.

UNDER the boundary conditions

$$\theta(0, t) = \theta_1 + \theta_2 \cos \omega t,$$

$$\theta(\infty, t) = 0,$$

the solution of the differential equation

$$\frac{\partial \theta}{\partial t} + \mu\theta = k \frac{\partial^2 \theta}{\partial x^2}, \tag{1}$$

expressing the linear flow of heat is

$$\theta(x, t) = \theta_1 e^{-p_1 x} + \theta_2 e^{-p_2 x} \cos (\omega t - qx + e), \tag{2}$$

in which

$$p_1 = \sqrt{\frac{\mu}{k}},$$

$$p_2 = \frac{1}{k}\sqrt{\frac{\mu + \sqrt{\mu^2 + \omega^2}}{2}},$$

$$q = \frac{1}{k}\sqrt{\frac{-\mu + \sqrt{\mu^2 + \omega^2}}{2}}.$$

In these equations k is the ratio of the thermal conductivity to the product of density and specific heat, and μ is a term giving the rate of radiation from the surface of the specimen considered. From the value of q it follows that the velocity of propagation of the heat wave represented by equation (2) is a function of the thermal conductivity, density, specific heat, the period $2\pi/\omega$ of the impressed temperature variation, and the rate of radiation from the surface of the rod. Of these five factors, the only uncertain one is the last; and this can be readily eliminated and

[1] The method of measuring heat conductivities which is described in the following pages is, it is thought, original. It however bears a certain resemblance to the methods of Ångström and Neumann, and it also appears from a recent article by T. Barratt (Proc. Phys. Soc. London, p. 347, 1914) that in some of the experimental features, the present method is similar to his. This however is a coincidence.

For an excellent summary of practically all the various methods of measuring thermal conductivities and for a very complete bibliography the reader is referred to Chwolson's Traité de Physique.

the conductivity found if the velocities corresponding to two separate periods are known. We have, calling v the velocity of propagation of the heat wave, λ its wave length and T its period,

$$v = \frac{\lambda}{T} = \frac{2\pi}{Tq},$$

whence

$$v^2 = \frac{8\pi^2 k}{T^2(-\mu + \sqrt{\mu^2 + \omega^2})}$$

or

$$\mu = \frac{\omega^2 v^2 T^2}{16\pi^2 k} - \frac{4\pi^2 k}{T^2 v^2}.$$

Hence, if, for a constant value of μ, the velocities v_1, v_2 correspond to the periods T_1, T_2 respectively, then

$$\frac{\omega_1^2 v_1^2 T_1^2 - \omega_2^2 v_2^2 T_2^2}{16\pi^2} = 4\pi^2 k^2 \left(\frac{1}{T_1^2 v_1^2} - \frac{1}{T_2^2 v_2^2} \right)$$

and

$$k = \frac{1}{4\pi} \left(\frac{T_1^2 T_2^2 v_1^2 v_2^2 (v_1^2 - v_2^2)}{T_2 v_2^2 - T_1^2 v_1^2} \right)^{1/2}.$$

The following is an account of some experimental work undertaken with the idea of reproducing the conditions of the problem just outlined.

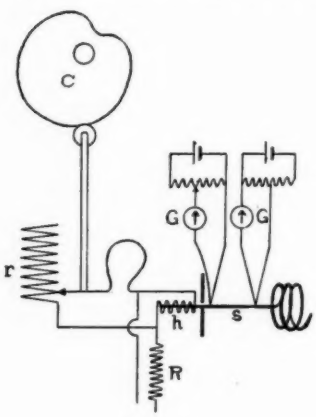

Fig. 1.

The essential parts of the apparatus are shown in Fig. 1. The specimen S is in the form of a wire, usually about 2.5 mm. in diameter and from 25 to 50 cm. in length. The greater portion of this wire is coiled up so that, all together, it occupies a space about 7 cm. long. One end of the specimen is inserted in a small resistance coil h which serves to supply heat to it. The coil h is connected in multiple with a variable resistance r, the two being attached through a larger resistance R to a steady source of current (in the present case, 110-volt D.-C. mains). The value of r is varied by the cam C in such a way that the current I through h is given by the expression

$$I = I_0 \left| \sin \frac{\omega}{2} t \right|.$$

(The design of the cam will be discussed in detail later.) To measure the velocities at which the impressed waves of heat travel along the speci-

men, two thermo-elements are attached a few centimeters apart to the end of the specimen at which heat is supplied. Each element is in series with a short-period galvanometer and a counter electromotive force equal to the mean value of the electromotive force of the element. By means of this arrangement, the motions of the galvanometers correspond to the variations of the temperatures of the points *a* and *b* from their respective mean temperatures; and to determine the velocity of wave propagation it is only necessary, knowing the distance between *a* and *b*, to determine by observation the time-lag between the motions of the galvanometers.

II. DETAILS OF THE APPARATUS.

The Specimen.—Exact dimensions of the specimens used will be found among the data. In the case of the specimen of tin, the thermo-junctions were between the tin itself and No. 40 constantan wire fused into its surface; and in the case of the copper the junctions were between the copper itself and No. 30 constantan wire inserted in very small holes in he copper and fastened with a solder of pure silver. For the purpose of checking the behavior of the apparatus, some specimens were equipped with three junctions about equally spaced, and with each of these it was found that the wave velocity between the first and second junctions was the same as that between the second and third.

The device found most convenient for holding the specimen and heating coil *h* is shown in Fig. 2. It consists of three circular disks of asbestos

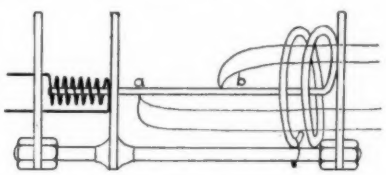

Fig. 2.

board fastened to a $\frac{1}{4}$-inch iron rod. The two end disks, in order to be readily removable, are clamped by nuts, and the middle one is cemented in place. The specimen is inserted in a hole just large enough to receive it in the central disk, and the opposite end is steadied by means of a bit of wire which ties it to the iron rod. The junction wires pass out through small holes in the right-hand disk. During use the whole arrangement is placed in a small wooden box, or wrapped with a sheet of paper, or inserted in an electric furnace in the cavity of which it just fits.

The Heating Coil h.—This coil, which is wound from No. 20 nichrome wire, has a diameter of 1 cm. and length of 2.5 cm. Its resistance at

room temperature is 1.5 ohms. In the writer's work the maximum current in h has been in the neighborhood of 2.5 amperes.

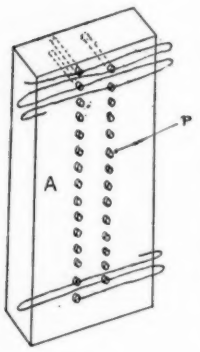

Fig. 3.

The Variable Resistance r.—This resistance is built up in fifty-two equal sections from No. 18 manganin wire, the manner of construction being shown in Fig. 3. A is a fiber block through which two columns of copper pins P are driven, and the wire, being arranged in a zig-zag form, is soldered to the ends of these pins which project from one face of A. The sliding contact which the cam actuates presses against the other ends of the pins. As a result of using a variable resistance of this kind, the current through the coil h, instead of varying continuously in agreement with the equation

$$I = I_0 \left| \sin \frac{\omega}{2} t \right|.$$

actually varies by a series of little jumps which approximate closely to the continuous value. However, the result is satisfactory, for with a period as great as eight minutes no lack of smoothness can be detected in the resultant heat-wave in the specimen. The form of this resultant wave has been tested at different times and been found to be strictly sinusoidal.

The Cam.—The cam must be of such a shape as to cause the temperature of h to vary in the manner specified by the first boundary condition. Call c the heat capacity of h, μ its radiation constant, and let $H = H(t)$ represent the rate of development of heat due to a current in h. Then the change of temperature of h is given by the equation

$$H(t)dt - \mu\theta dt = cd\theta,$$

and since θ must vary according to the relation

$$\theta = \theta_0(\alpha - \cos \omega t), \qquad \alpha \geqq 1,$$

we have

$$\begin{aligned} H(t) &= c\theta_0\omega \sin \omega t + \mu\theta_0(\alpha - \cos \omega t) \\ &= \theta_0[\mu\alpha - A \sin (\omega t + \varphi)], \end{aligned}$$

where

$$A = \sqrt{\mu^2 + c^2\omega^2}$$

and

$$\varphi = \tan^{-1}\left(-\frac{\mu}{c\omega}\right).$$

Dropping the phase angle φ and remembering that when $I = 0$, $H(t) = 0$, we may write

$$H(t) = A(1 - \cos \omega t)$$

$$= 2A \sin^2 \frac{\omega}{2} t,$$

and since $H(t)$ varies as the square of I,

$$I = \sqrt{2A} \sin \frac{\omega}{2} t,$$

or what, so far as the requirements of the present work are concerned, is equivalent,

$$I = \left| \sqrt{2A} \sin \frac{\omega}{2} t \right|.$$

To get such a current, consider two multiply-connected resistances, h and r, in series with a resistance R and a steady electromotive force. Assume the resistance R to be so large in comparison with h and r that, when h is held fixed and r is varied, the current I_0 supplied by the electromotive force is practically constant. Then, calling I the current through h,

$$I = \frac{I_0 r}{r + h} = a \left| \sin \frac{\omega}{2} t \right|$$

whence

$$r = \frac{ha \left| \sin \frac{\omega}{2} t \right|}{I_0 - a \left| \sin \frac{\omega}{2} t \right|}.$$

Or taking $a = \frac{1}{2} I_0$,

$$r = h \frac{\left| \sin \frac{\omega}{2} t \right|}{2 - \left| \sin \frac{\omega}{2} t \right|}.$$[1]

Since the value of r is proportional to the displacement of the sliding contact, the radii of the cam (except for an arbitrary additive constant) are given at once by this last expression. The writer has used for the values of ten radii equally spaced in an interval of 180°,

3.75 + 0.000 cm.	3.75 + 5.725 cm.
3.75 + 0.878 "	3.75 + 7.050 "
3.75 + 1.907 "	3.75 + 8.180 "
3.75 + 3.075 "	3.75 + 8.960 "
3.75 + 4.370 "	3.75 + 9.230 "

The cam was run by an electromotor operating through a reduction

[1] All of this work assumes that the changes of resistance of h and r due to changes of temperature are negligible, and apparently this assumption may be safely made.

gearing. The motor was run at as constant a voltage as possible and the change of period of the cam was secured by changing the ratio of the gearing.

The Galvanometers.—These instruments had sensitivities of 118 and 194 megohms, the more sensitive one being connected with the rear junction. They were critically damped, and having equal periods of 3.5 seconds, it was unnecessary to correct for their lag behind the electromotive forces of their respective junctions. They were arranged to cast images of a Nernst filament upon a scale five meters distant.

The time-lag of the rear galvanometer behind the leading one was determined by recording on a chronograph the times of transits of the images of the filament through three reference positions chosen near the center of the swings. The reason for using three positions rather than one will be clear upon referring to the data given upon page 443, for it will be noted that the three values of the time-lag determined during a single complete vibration often vary almost as much from one another as those taken in vibrations several minutes apart. With three reference positions it thus becomes possible to arrive at a given degree of accuracy in about one third the time that would be required when using but one. On the other hand, it was not found desirable to use more than three as the uncertainty incurred by having so many positions to watch counteracted the gain secured by the greater number of readings. The chronograph records were extended to include six to eight vibrations with a period of 130 seconds and from four to six vibrations with a period of 295 seconds. Longer records were not made because slow and irregular changes in the value of μ then had a noticeable tendency to affect the results.

The one condition, which above all others seems to be necessary to an accurate measurement of the time-lag, is that the mean temperature of the junctions remain constant, and until this condition has been attained it is of very little use to take readings.

More sensitive galvanometers than the two the writer has used would doubtless be desirable as it has been found necessary, with the present arrangement, to use a temperature variation at the leading junction of from 20 to 30 degrees Centigrade. Furthermore, with the rear galvanometer considerably more sensitive than the one used, it would be possible, not only to cut down the temperature amplitude, but also to place the junctions farther apart. This would of course increase the time lag and thereby increase the accuracy of the results. With the wires thus far used, damping of the heat wave has been considerable but not so great as was expected. The extent to which this damping occurs does not

of course affect the time lag between the galvanometers. Their motions are of the type known as forced vibrations and it can readily be shown that the time lag is therefore determined entirely by their free periods,

TABLE I.

Tin.

Diam. .25 cm. Length 25 cm. Distance between junctions 2.535 cm. Conductivity .1567.

Period.	Time-lag.			
	7.97	9.04	10.14	
	13.07	11.84	10.82	
	21.04	20.88	20.96	
	9.86	10.90	11.98	
	10.94	9.75	8.68	
133.0 secs	20.80	20.65	20.66	
	10.13	11.23	12.39	
	10.63	9.61	8.42	
	20.73	20.84	20.81	
	9.10	10.30	11.31	
	11.76	10.60	9.56	
	20.86	20.90	20.87	
	9.52	10.54	11.62	
	11.48	10.35	9.26	
	21.00	20.89	20.88	
	9.60	10.79	11.92	
	11.58	10.43	9.33	
	21.18	21.22	21.25	Average = 20.90 = 22.64 secs.
	12.43	13.50	14.37	
	14.08	13.00	11.97	
	26.51	26.50	26.34	
	13.28	14.23	15.19	
	13.34	12.20	11.19	
	26.62	26.43	26.38	
297.6 secs.				
	12.02	13.05	13.94	
	14.53	13.58	12.51	
	26.55	26.63	26.45	
	12.60	13.51	14.48	
	13.89	12.83	11.79	Average = 26.46
	26.49	26.34	26.27	= 28.66 secs.

the degree of their mechanical and electrical damping, and the period of the impressed electromotive force.

III. DATA AND RESULTS.

Table I. gives a sample set of readings as taken directly from a chronograph record. The three columns give respectively the time-lag (in an arbitrary scale) as determined from the transits occurring at the three reference positions. Motion from the left hand position (corresponding to the left hand column) to the right represents a cooling of the junctions.

Table II. gives in summarized form the results for copper and tin, the two metals upon which thus far accurate measurements have been made.

TABLE II.

Copper.

Diam. .25 cm. Length 30 cm. Distance between junctions 4.24 cm. Density 8.93.

Date.	Mean Temperature.	Specific Heat.	Period in Seconds.	Time-Lag in Seconds.	Conductivity.[1]	
Mar. 26	35°	.0928	130.7 293.0	11.77 15.52	...910	.9085 av.
Apr. 14	35	.0928	129.3 291.3	12.12 16.60	...907	
Apr. 16	60	.0938	129.8 293.0	12.26 16.97	...906	.9035 av.
Apr. 30	60	.0938	128.8 290.3	12.43 17.55	...901	

[1] Unit of conductivity is the calorie per centimeter per second per degree Centigrade.

Tin.

Diam. .25 cm. Length 25 cm. Distance between junctions 2.535 cm. Density 7.28. Mean temperature 35°. Specific Heat .0547.

Date.	Period in Seconds.	Time-Lag in Seconds.	Conductivity.	
Apr. 5	129.7 290.0	11.30 14.36	1551	
Apr. 12	130.1 294.5	11.41 14.66	1543	
Apr. 12	128.3 287.7	11.22 14.31	1553	
Apr. 12	133.0 297.6	11.32 14.33	1567	....1554 av.

The value of the temperature coefficient for copper as determined (only roughly of course) between the temperatures of 35° and 60° is —.00022, a value which agrees quite satisfactorily with a value found by Jäger and Diesselhorst[1] for one of their specimens of copper. The values

[1] Wiss. Abh. d. Phys. Techn. Reichsanstalt, 3, 269, 1900.

of the specific heat of copper which I have used are those of Naccari,[1] and of the specific heat of tin are those of Schimpff.[1] The values of the densities were taken from the *Tabellen* of Landolt and Börnstein.

Preliminary experiments indicate that the method here described may be advantageously applied to the measurement of the thermal conductivity at high and low temperatures and in a magnetic field, and more extensive measurements are now in progress. Work done with tellurium seems to show that the method will work satisfactorily for conductivities as low as .01.

CORNELL UNIVERSITY.

[1] Landolt u. Börnstein, Tabellen.

ON THE CORRECT FORMULA FOR THE DAMPING FACTOR IN HIGHLY DAMPED PERIODIC MOTION OF THE COIL OF THE D'ARSONVAL GALVANOMETER.

BY LINDLEY PYLE.

CONTRIBUTORS to the PHYSICAL REVIEW have discussed the moving-coil ballistic galvanometer in considerable detail from both the theoretical and experimental sides,[1] but a treatment, in this or any other publication, of the narrow limits of applicability of the commonly used formulae for the damping factor has not yet met the attention of the writer. It is becoming the practice to express the damping factor in the form, $(\theta_1/\theta_2)^{1/2}$, instead of in the less simple, but equivalent, logarithmic decrement form, $e^{\lambda/2}$.[2] To show that these expressions are often not even approximations to the truth, and to derive the true damping factor formula, it is necessary to recall certain equations.

The differential equation of motion of a d'Arsonval galvanometer coil is

$$Kd^2\theta/dt^2 + ad\theta/dt + b\theta = 0, \tag{1}$$

where the symbols have the usual significance. It follows that, when the damping is less than critical,

$$\theta = \frac{\omega_0}{s} e^{-\Delta t} \sin st, \tag{2}$$

where ω_0 represents the initial angular velocity when $t = 0$ and $\theta = 0$. $\Delta = a/2K$ and $s = (b/K - a^2/4K^2)^{1/2}$.

Let t_1 represent the time required to reach the first maximum value of θ (see Fig. 1). Then, by eq. (2),

$$2t_1/T = \frac{1}{\pi} \sin^{-1} (1 - a^2/4bK)^{1/2}, \tag{3}$$

where T represents the damped period. $2t_1/T = 1/2$ when $a = 0$, the ratio decreasing in magnitude with increase in a. It will be shown that this ratio is of vital importance in the evaluation of the damping factor,

[1] O. M. Stewart, PHYS. REV., Vol. 16, p. 158, 1903. Walter P. White, PHYS. REV., Vol. 23, p. 382, 1906. Anthony Zeleny, PHYS. REV., Vol. 23, p. 399, 1906. Paul E. Klopsteg, PHYS. REV., Vol. II., 2d Series, p. 390, 1913 and Vol. III., 2d Series, p. 121, 1914.

[2] Anthony Zeleny, loc. cit., p. 407.

i. e., the factor by which damped galvanometer throws must be multiplied to obtain the values of undamped throws.

If t_1 is the time required to reach the first positive maximum value of θ, say θ_1, then $t_1 + T/2$ is the time required to reach the first negative maximum value of θ, say θ_2 (see Fig. 1). Whence, by eq. (2),

$$\theta_1/\theta_2 = e^{\Delta T/2}. \tag{4}$$

Putting $a = 0$ in eq. (2), the *undamped* throw, θ_0, is given by

$$\theta_0 = \omega_0/(b/K)^{1/2}. \tag{5}$$

It can be readily shown that

$$\theta_0 = e^{\Delta t_1}\theta_1. \tag{6}$$

Eliminating Δ between (4) and (6) one obtains

$$\theta_0 = (\theta_1/\theta_2)^{2t_1/T}\theta_1. \tag{7}$$

That is, if damping were to be eliminated, the throw, θ_0, that would

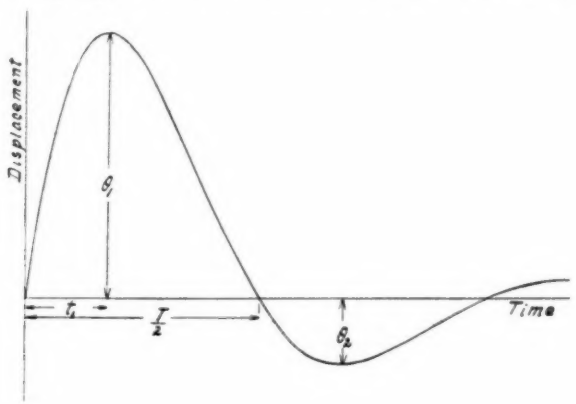

Fig. 1.

then be noted would be $(\theta_1/\theta_2)^{2t_1/T}$ times the observed first throw, θ_1. In short, $(\theta_1/\theta_2)^{2t_1/T}$ is the damping factor. It is customary to write the damping factor as $(\theta_1/\theta_2)^{1/2}$, justifying the expression by citing the constancy of the logarithmic decrement and stating that a *throw* corresponds to half (sic) a single swing. The error of approximation in doing this is not negligible unless the damping is small, a fact that has not been adequately pointed out in the literature of the subject. It is therefore of interest to calculate the ratio of the true damping factor to the commonly used damping factor for assigned values of θ_1/θ_2, making use of the following relation,

$$2t_1/T = \frac{1}{\pi}\tan^{-1}\pi/\log_e(\theta_1/\theta_2). \tag{8}$$

θ_1/θ_2 (Assigned).	$2t_1/T$ (Calculated).	$(\theta_1/\theta_2)^{2t_1/T} \div (\theta_1/\theta_2)^{\frac{1}{2}}$ (Calculated).
1	0.5	1
1.2	0.4815	0.9966
1.5	0.4592	0.9836
2	0.4309	0.9532
3	0.3929	0.8890
4	0.3677	0.8325
5	0.3493	0.7846
6	0.3351	0.7442
8	0.3139	0.6791
10	0.2987	0.6291

Fig. 2 shows a plot of columns one and three of the above table. The

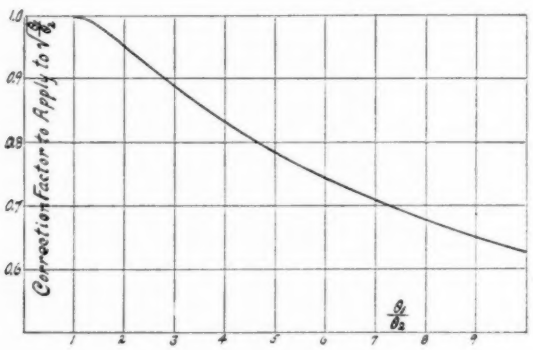

Fig. 2.

plotted curve shows at a glance, for any observed ratio of θ_1/θ_2, the true damping factor to be applied to the throw of *any* oscillating system whose motion is correctly defined by eq. (1). Thus, for $\theta_1/\theta_2 = 2.80$, the corresponding damping factor is 0.90 $(\theta_1/\theta_2)^{1/2}$.

The ratio of the damped period, T, to the undamped period, T_0, is given by

$$T/T_0 = (1 + (\log \theta_1/\theta_2)^2/\pi^2)^{1/2}, \qquad (9)$$

a formula in which appears the logarithmic decrement but not the damping factor; whence the evaluation of T/T_0 requires no reference to the curve of Fig. 2.

Though it is known that small amplitude oscillations of the suspended system in the moving-coil type of ballistic galvanometer follow very closely the type of motion indicated by (1),[1] it seems worth while to contribute the following comparison of observed and calculated data for the case of a Leeds & Northrup Type HB ballistic d'Arsonval galvanometer. Photographic records were obtained by reflecting a beam of light from the

[1] B. Osgood Peirce, Proc. Am. Acad., Vol. XLIV., p. 302, 1909.

mirror of the oscillating coil to a drum whose periphery, situated 50 centimeters from the mirror, was driven at a constant speed of approximately 7 millimeters per second. The coil was in a closed circuit of total resistance as indicated in the following table, and oscillations were set up by a loose-coupled inductive action of a neighboring circuit. Damping was varied by changing the resistance of the coil circuit. Fig. 1 is a fairly accurate copy, to actual scale, of one of the oscillograms obtained. See also a paper by B. Osgood Peirce.[1] The differences between

Circuit Resistance Ohms.	θ_1/θ_2 (Observed).	$T/2$ Sec.	t_1 Sec.	$2t_1/T$.	
				Observed.	Calculated.
20,000	1.556	6.23	2.90	0.466	0.456
7,000	2.083	6.31	2.79	0.442	0.452
5,000	2.554	6.45	2.68	0.416	0.408
4,000	3.017	6.49	2.60	0.400	0.393
3,000	4.145	6.76	2.47	0.366	0.366
2,000	9.072	7.53	2.32	0.308	0.305

the observed and calculated values of $2t_1/T$ are no greater than the observational errors associated with the reading of the oscillograms. Furthermore, $(1/T)$ log (θ_1/θ_2) plotted with $1/R$ yields a straight line, as would be expected from consideration of eq. (4) and the fact that that part of a due to the counter electromotive induced in the turns of the coil by its motion is proportional to the conductivity, $1/R$, of the closed circuit.

The curve of Fig. 2 seems to be of undoubted value in certain experiments involving highly damped oscillations of moving-coil ballistic galvanometers, in that the true corrective factor for damping may be read off without the troublesome, and often difficult, direct determination of the ratio of the time of throw of the coil to its period of oscillation.

Physical Laboratory,
Washington University,
St. Louis, Missouri,
July, 1915.

[1] B. Osgood Peirce, Proc. Am. Acad., Vol. XLII., p. 166, 1906.

ELECTRICAL OSCILLATIONS FROM MERCURY VAPOR TUBES.

BY BENJAMIN LIEBOWITZ.

I. INTRODUCTION.

SEVERAL methods for generating electrical oscillations by means of the mercury vapor arc have been proposed,[1] but the oscillations obtained by these methods are very limited in power or frequency, or are otherwise unsatisfactory. On the other hand, the natural instability and high sparking voltage of the mercury vapor arc would seem to adapt it to the production of powerful high-frequency currents by the Thomson (or Duddell) method. The following research was undertaken to investigate the possibility of obtaining oscillations in this manner. The experimental side of the investigation is confined mainly to mercury vapor tubes, but the theoretical side is generally applicable to arc oscillators in which the amplitude of the high-frequency current exceeds the supply current.

II. PRELIMINARY THEORETICAL CONSIDERATIONS AND EXPERIMENTS.

Arc Oscillations.

As is well known, there are three types of arc oscillations, usually referred to as first, second, and third kinds respectively. Nasmyth[2] has proposed the names "Duddell," "Poulsen," and "Wien" oscillations, which will be adopted here. In Duddell oscillations, the condenser current is smaller in amplitude than the direct current supplied; hence the total arc current is always greater than zero. Poulsen and Wien oscillations consist of condenser discharges of relatively large amplitude, alternating with periods of extinction of the arc. In Poulsen oscillations the condenser discharges are single unidirectional pulses, whereas in Wien oscillations the discharges are of the usual damped oscillatory form. Only the last two types of oscillations will be considered in this paper.

The voltage fluctuations across an oscillating arc give rise to periodic

[1] See, for example, Vreeland, PHYS. REV., 27, p. 286, Oct., 1908, and U. S. Patent Specification No. 1,079,250 (Lyle).

[2] PHYS. REV., Vol. 32, p. 69, Jan., 1911.

variations in the supply current. In practice, these variations are small, because a large series inductance is always inserted, hence the tendency has been to regard the supply current as constant. It will be shown later that these variations, however small, are of fundamental importance. A description of the cycle of events in arc oscillators, with emphasis on the pulsations in the supply current, is therefore in place.

A typical diagram of connections is shown in Fig. 1. E is the source of direct current, R_1R_2 are series resistances, L_1L_2 series inductances,

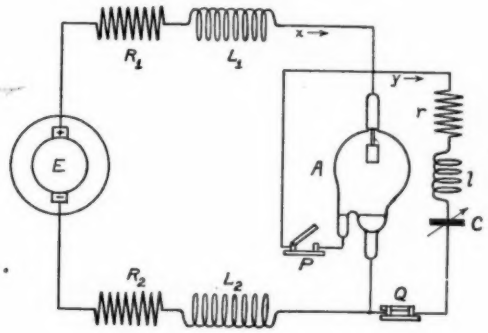

Fig. 1.

and A is any suitable arc, such as a Poulsen or Lepel arc. Condenser C, inductance l and resistance r comprise the oscillatory circuit, which is controlled by the switch Q.

Suppose that switch Q is open and that the direct current is flowing steadily through the arc. Upon closing Q the arc is extinguished, and the inductances L_1L_2 being to discharge their stored energy into the condenser; *i. e.*, the supply current decreases and the condenser voltage increases. This continues till C attains the sparking voltage, whereupon the arc becomes conductive, and allows the impressed E.M.F. to increase the supply current toward its original value. If the oscillations are of the Poulsen type, the arc remains conductive only long enough for the condenser circuit to execute one half swing, at the end of which the arc is again extinguished and the condenser is charged to a negative voltage smaller than the sparking potential. The energy left in the condenser is now returned to the supply circuit, and the supply current therefore continues to increase, till the condenser voltage is reduced to zero. This completes the Poulsen cycle, for energy now flows from the series inductances into the condenser until the sparking potential is reached again, etc. Hence, this cycle may be divided into three periods: (1) arc extinguished, condenser voltage increasing from zero to positive maximum, supply current decreasing; (2) arc conductive, condenser dis-

charging, supply current increasing due to action of impressed E.M.F.; (3) arc extinguished, condenser voltage increasing from negative maximum to zero, supply current increasing due to action of impressed E.M.F. and reversed condenser E.M.F.

In oscillations of the Wien type, as in the preceding case, the extinction of the arc is followed by a decrease in the supply current and a rise of the condenser voltage to the sparking potential; but the ensuing discharge is now of the usual damped oscillatory form. During the discharge, the arc is conductive and therefore allows the impressed E.M.F. to increase the supply current toward its original value. When the energy initially given to the condenser is practically all dissipated or radiated, the discharge ceases and the arc becomes non-conductive again. The condenser then receives energy from the series inductances till the sparking potential is reached, as before, and the cycle repeats itself. Hence this cycle may be divided into two stages: (1) arc extinguished, condenser voltage rising, supply current decreasing; (2) arc conductive, condenser discharging, supply current increasing due to action of impressed E.M.F.

It is readily seen that in both types of oscillations the decreases in the supply current must be greater than the increases during the first cycles, and therefore that the supply current must suffer a permanent reduction. But if a steady state of oscillation is to be reached, the increases in the supply current must eventually compensate for the decreases. Important consequences will be deduced from this in the mathematical section, but one result, which follows from elementary considerations, may be brought out here, viz., that it is the rate of energy dissipation in the oscillatory circuit, and not the sparking potential, which determines the magnitude of the impressed E.M.F. necessary for operation. For, no matter how high the sparking potential may be, it can always be attained by having a sufficiently large series inductance. On the other hand, the impressed E.M.F. has only a limited time in which to make good the decrease in current occasioned by the energy dissipation in each cycle, hence the greater the dissipation, the greater is the necessary impressed E.M.F. This result cannot be avoided by employing a large series inductance, for that affects the increases and decreases equally.

The fluctuations in the supply current have an important bearing also on the design of the series inductance. The functions of this inductance are to conduct the direct current to the arc, to charge the condenser to the sparking potential after each extinction of the arc, and to reflect the high-frequency energy produced so as to localize it in the oscillatory circuits. It is not generally appreciated that the last two functions are

of an essentially high-frequency character, and therefore will not be performed to the best advantage if the coils have much distributed capacity. For example, the time of propagation through the series inductance increases as the capacity of the windings is increased; but unless this time is short compared with the time required to raise the condenser to the sparking potential, only part of the coils will be useful in charging the condenser. Other disadvantages result from distributed capacity in the series inductance, which will be dealt with more fully after the mathematical developments have been presented.

Preliminary Experiments.

The foregoing considerations indicate that the successful utilization of the mercury vapor arc in a Thomson (or Duddell) circuit might possibly resolve itself into a question of merely putting a sufficiently large, properly constructed inductance in series with the tube. To test this possibility, a preliminary set of experiments was carried out.

The diagram of connections is that shown in Fig. 1. The condenser C in the oscillatory circuit was varied from 2×10^{-8} to 2×10^{-10} farads, approximately, the inductance l from the lowest values (inductance of the leads) up to about 0.006 henry. The switch P was used in starting the tube by tilting so as to make contact between the starting anode and the cathode in the usual manner. At times the starting anode was used as an operating anode, either alone or in conjunction with the regular anode. The supply voltage E was 240 in the first tests, but later this was raised to 500 and then to 750 volts. The supply current ranged from about 0.5 to about 2.5 amperes, but in most of the tests it was in the neighborhood of 1 ampere. The total value of the series inductance was approximately five henries. This was composed of ten coils arranged coaxially so as to get the greatest mutual and hence maximum total inductance. These coils had an inside diameter of 23.5 cm., an average outside diameter of 32 cm., and an average width of 5 cm. Some of the coils had 30 turns per layer, others had 45. Their D.C. resistances ranged from 6.6 to 40.0 ohms; their low frequency inductances from 0.124 to 0.778 henry. The wire in these coils was insulated with a double cotton covering. There was no extra insulation between layers. The coils having the least number of turns per layer were placed nearest the tube. All apparatus subjected to high voltage was carefully insulated.

The tubes experimented with varied from a small mercury quartz lamp to a rectifier bulb 25 cm. in diameter. In some of the tests the bulbs were cooled by immersing them in oil; in others they were warmed by first passing large currents through them; in others still, no attempts were made to control the temperature.

All efforts to generate steady oscillations in these experiments failed. Irregular pulsations of considerable amplitude, however, were readily obtained, as was shown by a hot-wire ammeter in the condenser circuit. When the system was oscillating in this irregular manner, sparking could be observed between the layers in the coils nearest the tube, but not in those more remote. This gave evidence not only of considerable energy dissipation in the coils, but also that the series inductance was not acting as a unit. It was concluded, therefore, that although the number of turns per layer was not very large, the coils used were not suitable for the purpose. Indeed subsequent measurements showed that at a frequency of 13,000 cycles the effective inductance of the individual coils was doubled and the effective resistance increased five hundred fold, thus proving that the distributed capacity of the windings is a factor of the greatest importance in such inductances.

It did not seem probable, however, that the series inductances or the rate of energy dissipation alone could be responsible for the negative results obtained. A mathematical investigation was therefore undertaken.

III. MATHEMATICAL DEVELOPMENT.

Assumptions, Notation, Etc.

In the oscillations under consideration, the transitions between conducting and non-conducting states of the arc, though rapid, occupy an appreciable time. Furthermore, when the arc is in the conducting state, its resistance varies with the current. These facts introduce mathematical difficulties, to avoid which we shall assume that the transitions between non-conducting and conducting states are instantaneous, and that the arc resistance, when finite, is constant. These assumptions, which are very similar to those adopted by Maisel[1] in his theory of the "Singing Arc," are certainly as justifiable as the corresponding assumptions usually made to avoid magnetic hysterisis and variable permeability.

The effect of these assumptions on the arc characteristic is indicated in Fig. 2. Here AB is a typical falling "static" characteristic, showing the relation between arc current and voltage when the current is slowly varied. DEC is a "dynamic" characteristic, showing the relation between arc current and voltage when the current is undergoing rapid cyclic variations. Under the above assumptions, the dynamic characteristic becomes the curve composed of the straight line OF and the voltage axis. These curves refer to Poulsen oscillations.

The oscillatory circuit of Fig. 1 is usually coupled with a secondary (antenna) circuit, but from the present point of view, the effect of this

[1] Phys. Zeit., 6, p. 38, 1905.

is mainly to modify the constants r and l of the primary oscillatory circuit. Hence, it is sufficient to confine the discussion to a system consisting of a supply circuit, an arc, and a single oscillatory circuit only (Fig. 1).

In practical cases, the amplitude of the oscillatory current is much larger than the supply current, and this will be assumed in the following. This assumption, while not necessary, simplifies the analysis by enabling an approximate value for the period of arc conduction to be readily found.

Although the periodic variations in the supply current are of fundamental importance, the mathematics is much simplified if these variations are kept small, as they generally are, by the use of a large series inductance. We shall assume, therefore, that the series inductance is large.

The following notation is used:

L = Total inductance of the supply circuit.
R = Total resistance of the supply circuit.
E = The impressed E.M.F. (assumed constant).
x = The Instantaneous value of the current in the supply circuit.
x_m = The mean value of the current in the supply circuit.
l = The inductance of the oscillatory circuit.
r = The resistance of the oscillatory circuit.
C = The capacity of the oscillatory circuit.
y = The instantaneous value of the current in the oscillatory circuit.
v = The instantaneous value of the condenser E.M.F.
q = The instantaneous value of the condenser charge: $q = Cv$.
v_0 = The breakdown voltage or sparking potential of the tube (or other arc).
q_0 = The charge on the condenser corresponding to v_0; $q_0 = Cv_0$.
t_1 = The time during which the tube (or arc) is in a conducting state.
t_0 = The time during which the tube (or arc) is not in a conducting state.
x_1 = The value of x at the end of time t_1.
x_0 = The value of x at the end of time t_0.

Similarly the subscripts $_1$ and $_0$ will be used to denote the values of other quantities at the end of time t_1 and t_0 respectively. Other symbols will be introduced where necessary. The complete period of a cycle is $t_1 + t_0$, hence the end of time t_0 is the beginning of t_1.

The system is looked upon as in a state of steady oscillation, and the investigation is made to ascertain under what conditions this state is first, a possible one, and second, a stable one.

Fundamental Condition for the Existence of Oscillations.

During the time t_1 the arc is in a conducting state, so that the supply circuit and the oscillatory circuit are both closed through the arc. The two circuits are practically independent of each other during this period, because all they have in common is the arc, and since the resistance of this is small, the coupling effect is small. Neglecting this small coupling, the following differential equations hold:

$$L\frac{dx}{dt} + Rx = E,$$

$$l\frac{d^2q}{dt^2} + r\frac{dq}{dt} + \frac{q}{C} = 0, \tag{1}$$

$$y = \frac{dq}{dt}$$

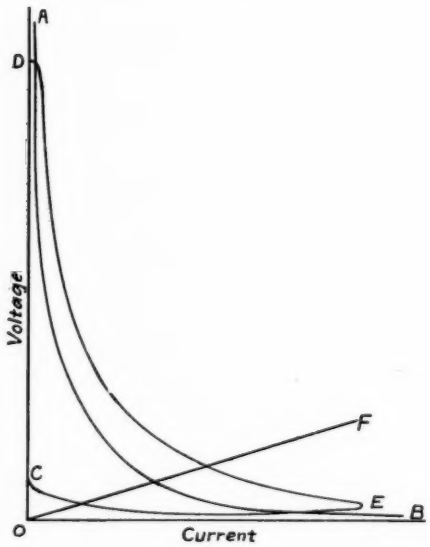

Fig. 2.

At the beginning of time t_1 the condenser voltage has its maximum value v_0, the charge corresponding to this is $q_0 = v_0C$, the oscillatory current is 0, and the supply current has its minimum value x_0. The initial conditions for the equations (1) are therefore:

$$x = x_0, \quad q = q_0, \quad y = 0. \tag{2}$$

Upon integrating equations (1), determining the constants of integration from (2), and neglecting terms multiplied by the small quantity δ_1/p_1, there results:

$$x = x_0 e^{-\frac{Rt}{L}} + X_0 (1 - e^{-\frac{Rt}{L}}),$$

(3) $$q = q_0 e^{-\delta_1 t} \cos p_1 t,$$

$$y = -p_1 q_0 e^{-\delta_1 t} \sin p_1 t,$$

where $X_0 = E/R$, $\delta_1 = r/2l$, and $p_1 = \sqrt{(1/lC) - \delta_1^2}$, or $p_1 = 1/\sqrt{lC}$, very nearly. These equations describe the course of events during the time t_1.

The total arc current is $x - y$, and this is 0 whenever the equation

(4) $$x_0 e^{-\frac{Rt}{L}} + X_0 (1 - e^{-\frac{Rt}{L}}) = -p_1 q_0 e^{-\delta_1 t} \sin p_1 t$$

is satisfied. If the oscillatory current is large in comparison with the supply current, as is assumed to be the case here, then (4) is satisfied approximately when

(5) $$t = \frac{\pi}{p_1}, \frac{2\pi}{p_1}, \frac{3\pi}{p_1}, \quad \cdots$$

After the arc current has passed through 0 for the nth time, the arc remains extinguished and period t_1 is brought to a close. Hence t_1 will be given approximately by

(6) $$t_1 = \frac{n\pi}{p_1}.$$

If $n = 1$, *i. e.*, if the arc remains extinguished after the arc current passes once through 0, then the oscillations are of the Poulsen type; if $n > 1$, the oscillations are of the Wien type. Mathematically, therefore, Poulsen oscillations are a special case of Wien oscillations.

At the instant $t = t_1$, then, the arc is extinguished, and the second stage of the cycle sets in. In this stage the supply and oscillatory circuits are connected in series so as to form a single circuit with resistance $R' = R + r$, inductance $L' = L + l$, and capacity C. In practical cases, however, r and l are very small compared with R and L; hence the differential equations which hold for the second stage may be written:

(7) $$L\frac{dx}{dt} + Rx + \int \frac{x\,dt}{C} = E,$$

or

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E,$$

and

$$x = \frac{dq}{dt}.$$

The initial conditions for these differential equations are the final conditions of the first stage of the cycle. Hence the required initial

conditions are obtained by substituting t_1 for t in equations (3) as follows:

$$
\begin{aligned}
x &= x_1 = x_0 e^{-\frac{Rt_1}{L}} + X_0(1 - e^{-\frac{Rt_1}{L}}),\\
q &= q_1 = q_0 e^{-\delta_1 t_1} \cos p_1 t_1,\\
y &= y_1 = x_1,\\
t_1 &= \frac{n\pi}{p_1} \text{(approximately)}.
\end{aligned} \tag{8}
$$

It is to be observed that q_1 may be positive or negative.

Upon integrating (7) and determining the constants of integration from (8), there results:

$$
\begin{aligned}
q &= -(Q - q_1)e^{-\delta_0 t} \cos p_0 t + \frac{1}{p_0}[x_1 - \delta_0(Q - q_1)]e^{-\delta_0 t}\sin p_0 t + Q,\\
x &= x_1 e^{-\delta_0 t} \cos p_0 t + \left\{ p_0(Q - q_1) - \frac{\delta_0}{p_0}[x_1 - \delta_0(Q - q_1)] \right\} e^{-\delta_0 t} \sin p_0 t,
\end{aligned} \tag{9}
$$

where $\delta_0 = R/2L$, $p_0 = \sqrt{(1/LC) - \delta_0^2}$ or $p_0 = 1/\sqrt{LC}$ approximately and $Q = EC$.

These equations represent a free oscillation of the supply and oscillatory circuits acting as one, with the initial conditions $x = x_1$ and $q = q_1$, together with the oscillation due to the presence of the impressed E.M.F., E. They hold from the time the tube is extinguished up to the time when the condenser voltage becomes v_0 and the arc breaks down again. This period, which has been denoted by t_0, is only a small part of a complete cycle of the combined supply and oscillatory circuits, provided that the series inductances are very large. For when this is the case, the voltage range from $\pm v_0 e^{-\delta_1 t_1}$ to $+ v_0$ is small in comparison with the total voltage range (approximately $2x_0\sqrt{L/C}$) of the free oscillation. It follows that $p_0 t$ remains small compared with $\pi/2$, and that $\delta_0 t$ is a small quantity (much smaller than $p_0 t$ in practice). Hence the following approximations are permissible:

$$
\begin{aligned}
e^{-\delta_0 t} &= 1 - \delta_0 t,\\
\sin p_0 t &= p_0 t,\\
\cos p_0 t &= 1 - \tfrac{1}{2} p_0^2 t^2.
\end{aligned}
$$

Upon substituting these values in equations (9) and rearranging, there results:

$$
\begin{aligned}
-q &= \tfrac{1}{2}\delta_0 p_0^2 (Q - q_1)t^3 - \{(\tfrac{1}{2}p_0^2 + \delta_0^2)(Q - q_1) - \delta_0 x_1\}t^2 - x_1 t - q_1,\\
x &= \tfrac{1}{2}\delta_0 p_0^2 x_1 t^3 - \{(\tfrac{1}{2}p_0^2 - \delta_0^2)x_1 + (\delta_0 p_0^2 + \delta_0^3)(Q - q_1)\}t^2\\
&\quad + \{(p_0^2 + \delta_0^2)(Q - q_1) - 2\delta_0 x_1\}\, t + x_1.
\end{aligned} \tag{10}
$$

At the end of time t_0, q returns to q_0 as v returns to v_0, and the cycle begins all over again. But a steady state of oscillation can exist only if x returns to x_0 when q returns to q_0, hence it may be stated as the fundamental condition for the existence of oscillations that the two equations

$$\begin{aligned} &\tfrac{1}{2}\delta_0 p_0^2(Q-q_1)t_0^3 - \{(\tfrac{1}{2}p_0^2+\delta_0^2)(Q-q_1) - \delta_0 x_1\}t_0^2 \\ &\qquad - x_1 t_0 + q_0 - q_1 = 0, \\ &\tfrac{1}{2}\delta_0 p_0^2 x_1 t_0^3 - \{(\tfrac{1}{2}p_0^2-\delta_0^2)x_1 + (\delta_0 p_0^2+\delta_0^3)(Q-q_1)\}t_0^2 \\ &\qquad + \{(p_0^2+\delta_0^2)(Q-q_1) - 2\delta_0 x_1\}t_0 + x_1 - x_0 = 0, \end{aligned} \tag{11}$$

shall have a positive real root in common. The equations (11) are obtained from (10) by substituting t_0, q_0, x_0 for t, q, x, respectively, and transposing. The fundamental condition for the existence of oscillations may also be stated as follows: a relation must exist among the coefficients of equations (11) such that the left-hand members shall have a real common factor. This relation can be found rigorously, but the result thus obtained is too complicated for ready physical interpretation. We proceed, therefore, by more approximate methods.

The value of $(x_1 - x_0)$, which occurs in the second of equations (11), may be found from the first of equations (8). The time t_1 is very short, since it is approximately equal to a small multiple of half the period of oscillation of the oscillatory circuit; and since the series inductance is large, the quantity Rt_1/L is small. Hence the approximation

$$e^{-\frac{Rt_1}{L}} = 1 - \frac{Rt_1}{L}$$

may be made with very little error. With this, the first of equations (8) gives:

$$x_1 - x_0 = (X_0 - x_0)\frac{KR}{L}t_0, \tag{12}$$

where $K = t_1/t_0$ is the ratio of time of discharge to time of charge of the condenser.

During the time t_0 the charge on the condenser is changed from q_1 to q_0, hence the supply current must deliver a charge $q_0 - q_1$ during the time t_0. Hence we may write

$$t_0 = \frac{q_0 - q_1}{x_m'}, \tag{13}$$

where x_m' is the mean value of the current during this time. Equation (13) is *a priori* evident, but it may also be regarded as resulting from the solution of the first of equations (11). For, if the latter be solved for t_0, the result is found to be very nearly equal to $(q_0 - q_1)/x_1$, under

the assumed conditions. Since equation (13) obviously gives the correct value for t_0, it follows that x_1 and x_m' must be very nearly equal. This proves mathematically that under the assumed conditions the fluctuations in the supply current are small.

Substituting (12) in the second of equations (11), dividing through by t_0, and then substituting (13) in the resulting equation gives:

$$\begin{aligned}(14)\quad & \tfrac{1}{2}\delta_0 p_0^2 \frac{x_1}{x_m'}\frac{(q_0-q_1)^2}{x_m'} - (\tfrac{1}{2}p_0^2 - \delta_0^2)\frac{x_1}{x_m'}(q_0-q_1) \\ & - (\delta_0 p_0^2 + \delta_0^3)(Q-q_1)\frac{(q_0-q_1)}{x_m'} + (p_0^2+\delta_0^2)(Q-q_1) \\ & - 2\delta_0 x_1 + (X_0 - x_0)\frac{KR}{L} = 0.\end{aligned}$$

We now make use of the fact that the supply current fluctuations are small, hence x_1/x_m' is a quantity very nearly equal to unity. Putting $x_1/x_m' = 1$, $x_1 x_m' = x_m^2$, and $x_0 x_m' = x_m^2$, where x_m is the mean value of x throughout the complete cycle, there results from (14) after multiplying through by $-x_m$:

$$\begin{aligned}(15)\quad & \left(2\delta_0 + \frac{KR}{L}\right)x_m^2 - \left\{(p_0^2+\delta_0^2)(Q-q_1) - (\tfrac{1}{2}p_0^2 - \delta_0^2)(q_0-q_1) \right. \\ & \left. + \frac{KR}{L}X_0\right\}x_m - \tfrac{1}{2}\delta_0 p_0^2 (q_0-q_1)^2 \\ & + (\delta_0 p_0^2 + \delta_0^3)(Q-q_1)(q_0-q_1) = 0.\end{aligned}$$

It is to be noted that although the approximations

$$\frac{x_1}{x_m'} = 1, \quad x_1 x_m' = x_0 x_m' = x_m^2$$

may be made in equation (14) without serious error, this does not mean that $x_1 - x_0$ may be neglected in the second of equations (11), for in the latter $x_1 - x_0$ is a quantity of the same order of magnitude as the other terms.

In practical cases, $R^2C/4L$ is small in comparison with unity, so that in the expressions $(p_0^2 + \delta_0^2)$ and $(\tfrac{1}{2}p_0^2 - \delta_0^2)$, the δ_0^2 may be neglected. Making this approximation, and remembering that $Q = EC$, $q_0 = v_0 C$, and $q_1 = v_1 C$, equation (15) reduces to:

$$\begin{aligned}(16)\quad & R(1+K)x_m^2 - \{E(1+K) - \tfrac{1}{2}(v_0+v_1)\}x_m \\ & + \frac{RC}{2L}(v_0-v_1)\{E - \tfrac{1}{2}(v_0+v_1)\} = 0.\end{aligned}$$

It can be shown that the last term may be neglected with little error, so that there results finally:

$$E = Rx_m + \frac{v_0 + v_1}{2(1 + K)},$$

(17) where $v_1 = v_0 e^{-\delta_1 t_1} \cos p_1 t_1 = v_0 e^{-n\pi \frac{\delta_1}{p_1}} \cos n\pi_1$ (approximately)

and $K = \frac{t_1}{t_0} = \frac{n\pi x_m}{v_0 - v_1}\sqrt{\frac{l}{C}}$ (approximately).

Equations (17) exhibit the relation in desired form. They show that due to the presence of steady oscillations, there is a reaction (or counter E.M.F.) in the supply circuit whose average value is given by

$$P = \frac{v_0 + v_1}{2(1 + K)}. \tag{18}$$

It is this reaction which causes the permanent reduction in the supply current when oscillations are set up, and which is measured by a D.C. voltmeter placed across the arc when it is oscillating. Since the fluctuations in the supply current are small, the product Px_m measures the average power delivered to the oscillatory circuit. As shown by equations (17), the greater the sparking potential, and the greater the damping of the oscillatory circuit (including the arc), and the smaller the ratio of the time of discharge to the time of charge, the greater is the reaction P. It should be noted that this ratio (K) is directly proportional to $\sqrt{l/C}$.

Equations (17) and (18) are general equations holding for all arc oscillations in which the oscillatory current is larger than the supply current. For the special case of Poulsen oscillations we have $n = 1$, in which case (17) and (18) become:

$$E = Rx_m + P_2, \tag{17a}$$

where

$$P_2 = v_0 \frac{1 - e^{-\frac{\pi\delta_1}{p_1}}}{2(1 + K_2)} \quad \text{and} \quad K_2 = \frac{\pi x_m}{v_0(1 + e^{-\frac{\pi\delta_1}{p_1}})}\sqrt{\frac{l}{C}}. \tag{18a}$$

In Wien oscillations, it usually happens that when the arc is extinguished after the n half-oscillations, the residual voltage v_1 is negligibly small. In this case we have:

$$E = Rx_m + P_3, \tag{17b}$$

where

$$P_3 = \frac{v_0}{2(1 + K_3)} \quad \text{and} \quad K_3 = \frac{n\pi x_m}{v_0}\sqrt{\frac{l}{C}}. \tag{18b}$$

The errors involved in the approximations employed in deducing equa-

tions (17) and (18) do not exceed a few per cent., provided that the series inductance is very large in comparison with the capacity of the oscillatory circuit. This condition is nearly always fulfilled in practice.

So far as energy relations are concerned, it follows from (17) that steady oscillations are *possible* when

$$E > P.$$

On the other hand, it will presently be shown that in order that the oscillations shall be *stable*, we must have:

$$x_m > \frac{1}{2} \cdot \frac{E}{R}. \tag{19}$$

From (17) and (19) it follows that for stable operation, we must have:

$$E > 2P, \tag{20}$$

i. e.,

$$E > \frac{v_0 + v_1}{1 + K}.$$

For Poulsen oscillations this becomes:

$$E > \frac{v_0}{1 + K_2}(1 - e^{-\frac{\pi \delta_1}{p_1}}), \tag{20a}$$

and for Wien oscillations:

$$E > \frac{v_0}{1 + K_3}. \tag{20b}$$

Applying these results to the preliminary experiments described above, suppose first that the system tended to produce Poulsen oscillations. The sparking potential v_0 was of the order of 10^4 volts, the factor $e^{-\delta_1 t_1}$ was probably 0.8 or less. The largest value of $\sqrt{l/C}$ was of the order of 10^3, which, with $x_m = 1$ in equation (17*a*) gives a rough value of 1/6 for K_2. Hence P was of the order of a thousand volts, and therefore a supply voltage greater than 2,000 would have been necessary to maintain Poulsen oscillations. If, however, the system tended to produce Wien oscillations (as it probably did), then P comes out somewhat greater than before, assuming that the condenser voltage is reduced to zero in about 10 half-oscillations. It is clear, therefore, that one reason for the negative results in the preliminary experiments is that the supply voltage of 750 was much too low.

STABILITY OF THE OSCILLATIONS AND EFFICIENCY.

We assume, as in the preceding section, that the variations in the supply current are kept small by the use of a large series inductance. The results to be obtained here are otherwise independent of the previous assump-

tions. One new assumption, however, is introduced, viz.,

$$Px_m = \text{constant},$$

where P and x_m have the same significance as before, but are now to be regarded as measured, rather than calculated, values. P and x_m are the ordinates and abscissæ of the arc characteristic "with oscillations," and experiment[1] shows that such characteristics have much the same shape as the ordinary "static" characteristic, *i. e.*, are approximately equilateral hyperbolas. As a result of experiment, therefore, we are justified in writing

$$\frac{d}{dx_m}(Px_m) = 0 \tag{21}$$

for at least small variations in x_m.

The average rate at which energy is supplied to the system is Ex_m; the average power dissipated in heat in the supply circuit is Rx_m^2, since the fluctuations in the supply current are small. Hence, denoting by U the average useful power delivered to the system, we have

$$U = Ex_m - Rx_m^2. \tag{22}$$

U represents useful power in the sense that it is power supplied to the oscillatory circuit. But the power drawn by the oscillatory circuit is Px_m, since the variations in x are small. Hence

$$Px_m = U. \tag{23}$$

This equation, which expresses the equilibrium between power supplied to and power drawn by the oscillatory circuit, can be true on the average only, owing to inevitable disturbances. If, in a departure from this equilibrium condition, x_m is decreased, and if U is thereby made larger than Px_m, then the oscillations are stable; but if U is thereby made smaller than Px_m, the oscillations are unstable. Hence the fundamental condition for stability is:

$$-\frac{\partial U}{\partial x_m} > -\frac{\partial}{\partial x_m}(Px_m). \tag{24}$$

Using (21), this becomes

$$\frac{\partial U}{\partial x_m} < 0. \tag{25}$$

From (22), U is a maximum when

$$\frac{d}{dx_m}(Ex_m - Rx_m^2) = 0,$$

i. e., when

$$x_m = \frac{1}{2}\frac{E}{R} = \bar{x}_m. \tag{26}$$

[1] See Nasmyth, loc. cit.

When $x_m < \bar{x}_m$, $\partial U/\partial x_m$ is positive, and when $x_m > \bar{x}_m$, $\partial U/\partial x_m$ is negative. From (25) it follows that the oscillations will be stable only when $x_m > \bar{x}_m$, *i. e.*, when

$$x_m > \frac{1}{2}\frac{E}{R}, \tag{19}$$

which is the relation used in the preceding section to find the impressed E.M.F. necessary for operation.

The condition (19) imposes a limitation on the efficiency of arc oscillation generators. For if η denote the efficiency of the supply circuit, then

$$\eta = \frac{P}{E} = \frac{P}{Rx_m + P}, \tag{27}$$

since $E = Rx_m + P$. From (19) and (27) there results

$$\eta < \tfrac{1}{2}. \tag{28}$$

We have, therefore, the following theorem: In arc oscillation generators supplied with energy from a constant potential source, at least half of the total energy input must be dissipated in heat in the supply circuit.

It should be noted that this theorem refers to the supply circuit only, and says nothing about the efficiency of the arc itself. It should also be noted that the theorem assumes nothing except that Px_m is constant for small variations in x_m and that the pulsations in the supply current are small. Increasing the pulsations in the supply current obviously cannot improve matters, so that the last assumption involves no loss of generality.

The above theorem was first announced by the author in a paper entitled "Preliminary Note on a Mercury Vapor Tube Oscillator," read before the American Physical Society.[1] The detailed proof, however, was not given.

This limitation of the efficiency of arc oscillation generators is serious, but it can be removed (theoretically, at least) by a suitable choice of the source of supply. Suppose, for example, that the impressed force, instead of being constant, falls off linearly with increasing current. Then, writing $(E - hx_m)$ in place of E, equation (22) becomes

$$U = Ex_m - (R + h)x_m^2. \tag{22'}$$

U now has its maximum when

$$x_m = \frac{E}{2(R + h)} = \bar{x}_m, \tag{26'}$$

so that the condition for stability becomes:

$$x_m > \frac{E}{2(R + h)}, \tag{19'}$$

[1] PHYS. REV., March, 1915, Vol. 5, Second Series.

i. e., the critical current $\bar{x}_m$ is shifted to lower values. Since we now have

$$E - hx_m = Rx_m + P, \tag{17'}$$

the expression for the efficiency is

$$\eta = \frac{P}{E - hx_m} = \frac{P}{Rx_m + P}, \tag{27'}$$

which is the same as (27). From (19') and (17') it is clear that Rx_m can be made small in comparison with P by making h large in comparison with R. By this means the efficiency of the supply circuit can be made high. All that is necessary is a source in which the E.M.F. falls off very rapidly with increase of current, *i. e.*, a source which approaches the characteristics of a constant current supply, rather than a constant potential supply. But there is a practical difficulty in the way, viz., the voltage changes must follow the current changes almost instantaneously. For if the arc current goes below the critical value $\bar{x}_m$, a very short time is sufficient for extinction. Thus, for example, if a high-voltage, differentially compounded D.C. generator were employed, the magnetic changes following the current changes in the series field would be much too slow, ordinarily, to be very useful here.

Although the mathematical developments contained herein are original with the author, so far as he knows, the suggestion of using a source whose E.M.F. decreases with increase of current is not new. Chaffee,[1] in giving the efficiency of his arc, omitted the losses in the series resistances, on the ground that they could be done away with by the use of a proper generator. But no explanation was given, and the practical difficulty emphasized above was not mentioned.

It may be added that any device which shifts the maximum value of U to lower values of the supply current will raise the theoretical efficiency limit; but to be effective, the device must be very rapid in action.

From the point of view of the subsequent experiments with mercury vapor arcs, the most important results of the mathematical development are: (1) The impressed E.M.F. must be large (several thousand volts), on account of the large value of P; (2) the series inductance must be large (several henries) and must be constructed so as to act as a unit for the rapid fluctuations of current and voltage; (3) in the oscillatory circuit, the ratio l/C should be as large as possible.

[1] Amer. Acad. Proc., 47–9, p. 267, 1911.

IV. SUBSEQUENT EXPERIMENTS WITH MERCURY VAPOR TUBES IN DUDDELL CIRCUITS.

The Series Inductance.

The important part played by distributed capacity in the series inductance was mentioned in an earlier section, but the matter requires further attention. Suppose the turns in the coil to be numbered 1, 2, $\cdots p$, $\cdots q \cdots n$, in the other of winding. The energy stored in the dielectric between any two turns p and q at any instant is $\frac{1}{2}C_{pq}V_{pq}^2$, where C_{pq} is the capacity and V_{pq} is the potential difference between those turns. The total electrostatic energy stored in the coil is therefore $\frac{1}{2}\Sigma C_{pq} \cdot V_{pq}^2$, the summation being extended to include every pair of turns in the coil. If the fall of potential through the windings were uniform, V_{pq} would be directly proportional to $(q - p)$. In general, it is clear that the greater $(q - p)$, the larger is V_{pq}. In a single layer coil, as V_{pq} increases, C_{pq} decreases, so that none of the terms in the summation can become large. In a multiple layer coil, however, pairs of turns for which $(q - p)$ is large are brought close together, with the result that the electrostatic energy stored in the coil can well become as large as, or even much larger than, the energy stored in the condenser in the oscillatory circuit. This has several important disadvantages: first, the time of propagation through the coil is increased, so that only part of the coil is really useful; second, instead of a single oscillatory circuit with one degree of freedom, or a pair of coupled circuits with two degrees of freedom, we have a very complicated system with many degrees of freedom, which materially affects the frequency of the oscillatory circuit (or circuits); third, the "choke-coil" effect is very much reduced by the ease with which high frequency energy is propagated through such coils, whereby the protection to the supply is weakened and energy wasted; fourth, oscillations are set up locally in the coils, which involve an additional waste of energy. This is a very important matter, but a more thorough treatment cannot be given here. It may be pointed out, for example, that as a consequence of these considerations, doubt is thrown on the accuracy of arc frequency determinations in which multiple layer series inductances have been employed.

It may be mentioned in this connection that Wills[1] found that the minimum value to which the current through a mercury vapor tube could be reduced without extinguishing the arc depended not only on the magnitude, but also on the *form* of the inductance in series with the tube. Since the distributed capacity and inductance of the choke coils and of

[1] PHYS. REV., Aug., 1904.

the leads act as an oscillatory circuit shunted across the tube, this observation is evidently in accord with the above discussion.

These considerations show that the series inductance should be made up of single-layer coils; or, if multiple layer coils are used, the separation between the layers should be proportional to the number of turns per layer. To meet these requirements, a large inductance (see Fig. 3) was

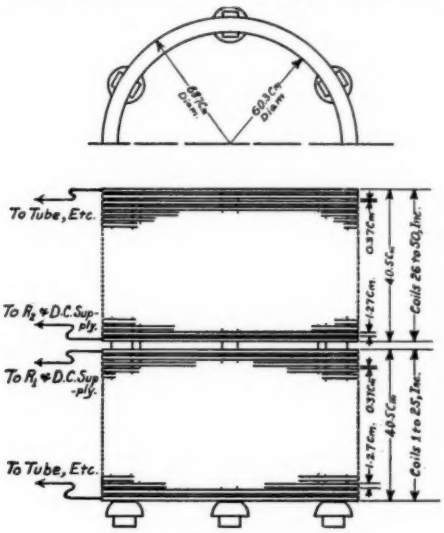

Fig. 3.

constructed of fifty single-layer spirals, each consisting of eighty turns of copper ribbon 1.27 × 0.0254 cm. in section, the turns being insulated by jute paper ribbon of the same section. The inside diameter of the spirals was 60.3 cm., the outside diameter 68.7 cm. The coils were tested individually at 300 volts and 500 cycles, and were then arranged in a pile, as shown in Fig. 3, and connected in series. The separation between spirals averaged 0.38 cm., except for the two middle ones, which were separated about 2 cm. The complete inductance had a coefficient (measured at 60 cycles) close to five henries, and a resistance to direct current of 41.3 ohms.

A narrower ribbon than 1.27 cm. would have been preferable, but a preliminary calculation showed that the coils ought to be satisfactory for arc frequencies up to 200,000. In order to make sure, however, that the spirals did have too much capacity, one of the coils was subjected to careful wave-meter and bridge measurements. The wave-meter results were 220,000 and 1,400,000 cycles for the natural frequency of the coil alone, and 54,500 cycles for the natural frequency of the coil connected to a condenser of 0.001 mfd. capacity. From these values the equivalent

concentrated inductance and capacity of the spiral were calculated to be 0.0080 henries and 0.000065 mfd. respectively, for frequencies not exceeding 220,000. The bridge measurements gave for the effective inductance and resistance of the coil 0.00846 henries and 7 ohms at 13,710 cycles, and 0.00834 henries and 4 ohms at 7,690 cycles. From these results the true inductance was calculated to be 0.00828 henries, which value is probably more correct than the former.

These measurements indicated that the spirals would be satisfactory for a series inductance for arc frequencies up to 100,000 cycles. This was corroborated by the behavior of the inductance in the experiments.

Repetition of First Experiments.

With this new inductance and with an impressed E.M.F. of 4,000 volts obtained from an old motor-generator set, the preliminary experiments described in Section II were repeated. The connections were as before (Fig. 1). The leads from the resistances R_1R_2 (lamps in series-parallel) were connected to terminals of the two middle coils of the inductance, and the tube was connected to the free terminals of the first and last coils. In this way the best voltage distribution throughout the whole inductance and an equal division of the inductance between both sides of this line were obtained.

Variously shaped tubes, different temperatures and different ratios of inductance to capacity were tried, but the results were not much better than before. The most plausible explanation of these negative results seems to be that the damping due to losses in the tube is extremely high. It is quite likely, if the system tended to produce Wien oscillations, as it probably did, that each wave-train lasted only for five or six half-swings, or less. Damping due to spark-gaps or arcs may easily be as large as this; *e. g.*, see Chaffee's[1] oscillograms. Assuming such a short wave-train, the minimum impressed E.M.F. necessary for operation was calculated from equations (17 *b*) and (20 *b*) to be greater than 5,000 volts. The only hope of obtaining oscillations with the available equipment, therefore, lay in decreasing the damping.

The ratio of inductance to capacity in the oscillatory circuit plays a very prominent part in this connection. It controls the damping in accordance with the ordinary laws, and it affects the factor K, as shown by equations (17). But most important of all, perhaps, is its effect on the initial amplitude of the discharge, which, because of the high sparking voltage, tends to very large values. This means that mercury vapor under low pressure, ionized to carry a moderate current, is suddenly

[1] Loc. cit.

called upon to conduct a current many times larger. The principal dissipation of energy in the tube may well lie in the creation of the necessary new ionization at the beginning of each cycle. Increasing the ratio of inductance to capacity is therefore beneficial for three reasons, and special efforts were made to make this ratio sufficiently large. Up to the present time these efforts have not been very successful, but the writer is still of the opinion that steady oscillations can be obtained in this manner. Experiments along the present lines have been discontinued for the time being, because of the discovery of a more promising method.

V. A DOUBLE-CATHODE OSCILLATOR.

Stability of Parallel Mercury Arcs.

In the course of the preceding experiments attempts were made to lower the damping in the tube by passing two parallel discharges, each from its own cathode to its own anode or to a common anode, simultaneously through the tube. This led to the discovery that two such parallel discharges are much more unstable than a single discharge. Thus, in one instance, when a capacity of 0.002 mfd. was necessary to extinguish the single discharge, a capacity of less than 0.0002 mfd. was sufficient to extinguish one of the parallel discharges. It became evident at once that here was a means of increasing the ratio l/C to a marked extent. An investigation of this matter resulted in the double-cathode oscillator which was briefly described in the paper entitled[1] "Preliminary Note on a Mercury Vapor Tube Oscillator." A more detailed description will now be given here.

Experimental Arrangement and Theoretical Considerations.

The arrangement of the apparatus is shown diagrammatically in Fig. 4. A mercury vapor tube T is provided with two mercury cathodes K_1 and K_2, and an iron or graphite anode A. The cathodes are connected to the negative terminal of the high-voltage generator E through their respective inductances L_1 and L_2 and resistances R_1 and R_2. The anode is connected to the positive terminal of E through the inductance L_3. Between the cathodes is connected the oscillatory circuit consisting of inductance l, condenser C, resistance r, and switch Q.

To facilitate the starting of the discharges, the switch P is provided, which, if thrown to the right, connects cathode K_2 to inductance L_2, and if thrown to the left, connects K_2 to anode A. When the switch is in the latter position, K_2 is an anode; the discharge from cathode K_1 can therefore be started by tilting the tube in the usual manner. After this

[1] Loc. cit.

has been done, the switch P is thrown to the right and the tube is again tilted, which starts the discharge through K_2 without extinguishing that through K_1. During these initial operations, the switch Q controlling the oscillatory circuit should be kept open. If the inductances L_1 and L_2 are sufficiently large, and if they have not too much distributed capacity, the parallel discharges between the two cathodes and the common anode are very stable.

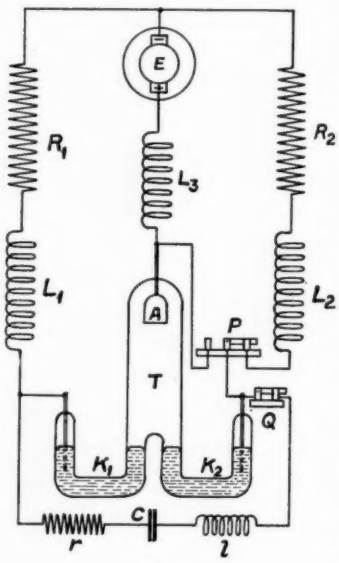

Fig. 4.

When the switch Q is closed, the stability of the parallel discharges is destroyed, and, if the necessary conditions are fulfilled, steady oscillations are obtained. As in the previous case, the oscillations may be divided into two classes, one analogous to Poulsen's, the other to Wien's. The former will be considered first.

Upon closing switch Q, then, the parallel discharges become unstable and one of them, say that through K_2, is extinguished. The current through L_2 must now traverse the oscillatory circuit, and in consequence must decrease while the condenser voltage increases. This continues till the sparking potential is attained, whereupon the vapor between the cathodes breaks down and the circuit Clr begins a damped oscillatory discharge. Simultaneously, K_1 becomes an anode and cathode K_2 becomes operative again. The current through L_2 is now restored to its original path, and therefore begins to increase toward its original value, due to the action of the impressed E.M.F. The current through L_1, on the other hand, must now traverse the oscillatory circuit; hence, while the first half-swing is in progress, there is a small energy transfer between L_1 and C. During the first quarter-period, the direction of this energy flow is from C to L_1, during the second quarter-period from L_1 to C. At the end of the half-swing, the condenser is charged to a reversed voltage smaller than the sparking potential. In oscillations analogous to Poulsen's, which we are now considering, this voltage is insufficient to start the reverse discharge, hence C continues to receive energy from L_1, whose current therefore decreases, till the sparking potential is reached. Thereupon the tube breaks down again, K_2 becomes an anode and K_1 a cathode. The current through L_1 is thereby restored to its original path and therefore begins to increase due to the

action of the impressed E.M.F., but the current through L_2 must again traverse the oscillatory circuit. During the first part of the reverse discharge, there is a small energy flow from C to L_2; during the second part, from L_2 to C. At the end of the half-swing the condenser is charged to a voltage smaller than the sparking potential, hence C continues to receive energy from L_2, whose current therefore continues to decrease, till the sparking potential is reached again. The cycle then repeats itself.

We see that in each cycle there is a period of increase in the current through L_1, a period of practically no change, and then a period of decrease. The variations in the current through L_2 are entirely similar. During the first cycles, the decreases in the supply current must be greater than the increases, and the supply current therefore suffers a permanent diminution. But ultimately, equilibrium between the decreases and increases will obtain, when the supply current has been reduced to some mean value x_m greater than the critical value $\bar{x}_m$, provided that the necessary conditions are satisfied. These conditions may be found by a mathematical analysis similar to that which has been given for the previous oscillators. The computation has not been carried out as yet, but it is evident that the results must be of the same general character as those already obtained.

The oscillations described are of nearly pure sine shape; they are therefore analogous to Poulsen's only in that the sparking potential is the same for each half-swing. The sparking potential, however, need not be the same for each half-oscillation, for the first rush of current may ionize the vapor so highly as to permit several successive discharges to pass before the tube regains its sparking potential. We have, therefore, a new kind of oscillations, which are analogous to Wien's. In other respects these oscillations are the same as the preceding, for the extinction of one cathode and the starting of the other at the end of each half-swing must take place as before. The essential difference between the two types of oscillations is, then, that in one the amplitude is constant, whereas in the other it decreases and increases periodically. It should be noted, also, that the frequencies are not exactly the same in both cases.

Kind of Oscillations Generated.

Experimentally, the oscillations generated were always of variable amplitude, and the frequency of the amplitude variations was above audibility. This was determined by coupling the oscillatory circuit very loosely to an Armstrong receiver[1] and observing the character of

[1] U. S. Pat. Spec. No. 1, 113, 149.

the sound produced. If the oscillations had been of constant amplitude, a clear note would have been generated in this receiver; actually, however, such a note was obtained but rarely, and then for short intervals of time only. Upon changing the receiver over into the ordinary audion type, the sound produced was a steady hiss, indicating that the frequency of the variations in amplitude was above audibility. This is no proof, of course, that the oscillations generated were of the character described as analogous to Wien's, but the readings of the ammeter in the oscillatory circuit were so steady, and the character of the hiss produced was so regular, that there can be little doubt that the oscillations themselves were very regular. Additional evidence of the regularity of the oscillations is furnished by the occasional production of a fairly clear note in the Armstrong receiver and by the fact that the switch Q (Fig. 4) could be opened so as to draw a high-frequency arc several millimeters long without disturbing the steadiness of the ammeter reading. It seems safe to conclude, therefore, that the oscillations generated were of the character described as analogous to Wien's. The writer hopes that this will receive experimental verification by Braun tube oscillograms.

Attempts to produce oscillations of constant amplitude were progressing, and it seemed very probable that the desired result would be achieved; but an accident to one of the high voltage generators prevented further efforts along these lines.

Experimental Details.

When the experiments with the double-cathode oscillator were first started, the same difficulties were encountered as in the previous cases. Currents of large amplitude and very high frequencies were easily obtained, but they were almost as irregular as those first obtained. The constants of the oscillatory circuit were varied through a wide range, but it was not until a coil having an inductance of 0.021 henries was inserted in the circuit that very steady oscillations were obtained. Furthermore, it was usually found necessary to warm the tube before attempting to make it oscillate. The range of capacities employed was from 0.002 to 0.0002 mfd. approximately, hence the ratio l/C varied between 10^7 and 10^8; these are extremely high values. But large ratios of l/C cause corresponding reductions in the oscillatory currents, which, in consequence, did not exceed 1.25 amperes (R.M.S. value) in the present experiments. This does not necessarily mean a small available high-frequency current, however, for by closely coupling the oscillatory circuit with another having a much smaller ratio of l/C, the available current can be multiplied many times.

A few additional experimental details are worthy of note. When steady oscillations were first obtained, an impressed E.M.F. of 4,000 volts was used. But even after the accident to one of the generators made it necessary to reduce this voltage to 2,000, steady oscillations could still be obtained, although the adjustments were more critical, of course, and further efforts to obtain oscillations of constant amplitude had to be abandoned. The supply current in these experiments ranged between 0.3 and 0.5 ampere, through each cathode. For resistances R_1R_2 (see Fig. 4), lamps in series parallel were employed. The inductances L_1 and L_2 consisted each of twenty-two of the spirals described in Section IV, the six remaining spirals being used for the inductance L_3. The coefficients were 1.73, 1.71 and 0.21 henry respectively. The use of the six spirals for the inductance L_3 was not resorted to until the generator connected to the anode A was injured, repaired, and then injured again so that it could not be repaired. The cause of these disasters was finally traced to the propagation of high-frequency energy through the multiple-layer coil which was then employed for the inductance L_3. There had seemed to be no reason to expect such propagation from the anode, so no special precautions had been taken to protect the generator to which it was connected. When the true state of affairs was realized, the multiple layer coil was replaced by the six spirals referred to, and thereafter no trouble was experienced from that source. This is a striking demonstration of the relative merits of single-layer and multiple-layer coils used as series inductances, choke-coils, etc., in high frequency work.

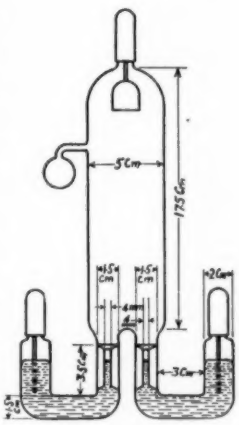

Fig. 5.

In order to determine the most suitable form of tube for the double-cathode oscillator, several different forms were tried. Fig. 5 is a drawing of the tube which gave the most consistent results. It will be seen that the cathodes in this tube are constricted to 4 mm. diameter. This was done, originally, in the effort to lower the sparking potential, but the amount by which the sparking potential was thereby reduced, if at all, was too small to be of much benefit. On the other hand, constricting the cathodes seemed to aid in steadying the action of the tube, although sufficient experiments have not as yet been carried out to ascertain definitely to what extent such constriction is necessary or desirable. Complications in the experiments are introduced by the facts that the temperature of the tube plays a very important part, and that this in

turn is affected by the volume of the tube, unless the temperature is controlled by external means.

Frequency Measurements.

It is not the purpose of this paper to enter into the much discussed topic of the frequency of arc oscillations further than to give the results of measurements on the frequency of the present oscillator. These measurements did not go above 80,000 cycles per second, because regular oscillations could not be obtained with an inductance much smaller than 0.020 henry. The lower limit of the measurements was 50,000 cycles, as this was the lowest value for which the wave-meter was calibrated, hence the frequency observations are confined to a rather narrow range. With an inductance of 0.0210 henry, the capacity of the oscillatory circuit was varied and observations were made at arc frequencies of 80,000, 72,800, 65,300, 59,800 and 50,400 cycles per second. The oscillatory circuit was then completely disconnected from the arc, and excited by means of a buzzer. In this way the natural frequencies were determined for the same capacity values as in the preceding measurements, and were then compared directly with the arc frequencies. The results showed that at 80,000 cycles the two frequencies differed by less than the experimental error, that at 50,400 cycles the arc frequency was approximately 2 per cent. lower than the natural frequency, and that between these limits the departure from the natural frequency steadily decreased as the frequency was increased. These departures from the Thomson frequency are relatively very small.

Power and Efficiency.

Owing to its high sparking potential, the mercury vapor tube as an oscillator is best adapted for large powers. But because of the limited output of the high-voltage set, no attempt was made to obtain much power in the present experiments. The resistance of the oscillatory circuit was 33 ohms, so that the actual high-frequency output was 50 or 60 watts. But although its power and efficiency have been very low up to the present time, the general behavior of the oscillator indicates that it is excellently adapted for very large powers.

Comparison with Duddell's Parallel Arc Method.

At first sight it would seem that the double-cathode method described here is but a slight modification of the parallel arc oscillator of Duddell[1] and of Nasmyth.[2] Further considerations show, however, that such is

[1] Inst. Electrical Engineers' Journ., 30, Feb., 1901.

[2] PHYS. REV., Vol. 28, p. 459, 1909.

not the case. The most important difference between the two oscillators lies in the fact that in the double cathode method the oscillatory current passes directly between the cathodes, the anode carrying the direct current discharge only, whereas in the parallel arc method the oscillatory current must pass from the cathode to the anode of one arc and thence from the anode to the cathode of the other arc. For this reason the parallel arc method seems to be suitable for Duddell oscillations only, but no such limitation applies to the double cathode method, as has been shown. A further difference, which depends on the character of the arc itself rather than on the system of circuits in which the arc is used, was brought to light by some simple experiments. Three solid carbon rods, arranged as shown in Fig. 6, were connected through large inductances and suitable resistances to the 220-volt supply in such a way as to make one carbon an anode (or a cathode) and the other two cathodes (or anodes). Between the latter a large condenser was connected. By manipulating the carbons, the arcs could be struck so as to start from separate positive craters and remain independent throughout their entire lengths, as shown in Fig. 6, *A*: or they could be struck so as to start from a common positive crater and remain common throughout a considerable part of their lengths, as shown in Fig. 6, *B*. Case *A* is the parallel arc method, case *B* is the double-cathode method. With about 4 amperes flowing through each arc, a capacity of 10 mfd. was always more than sufficient to give violent oscillations in Case *A*, whereas with the same current in case *B*, a capacity of 50 mfd. at times gave no oscillations, at other times only weak ones, depending on the length of arc in common. We see, therefore, that merging two parallel carbon arcs for a considerable part of their lengths markedly decreases the instability, whereas in parallel mercury vapor arcs, the length of positive columns in common has no influence on the instability. This shows that in the carbon arc the seat of the instability is in the ionized column, while in the mercury vapor arc the seat is at or near the cathode bright spot.

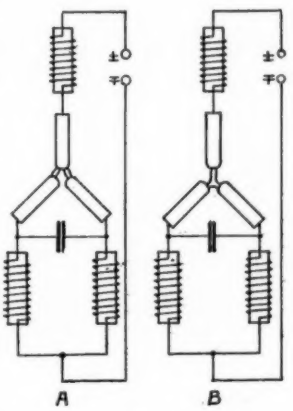

Fig. 6.

Attempts to apply the double-cathode method to other than the mercury vapor and carbon arcs have not as yet been made. Such attempts would be worth while, perhaps, if only for the light which would be thrown on the seat of the instability in the various arcs.

VI. SUMMARY.

A qualitative outline of the theory of arc oscillation generators is given, and the possibility of utilizing the mercury vapor arc in a Duddell or Thomson circuit is discussed. Preliminary experiments with mercury vapor tubes in a Duddell circuit are described, in which a maximum supply voltage of 750 and a series inductance, consisting of a number of multiple-layer air-core coils and amounting to 5 or more henries, were employed. All efforts to obtain steady oscillations failed.

The negative result is partly, but not entirely accounted for by the form of series inductance used. A mathematical investigation is therefore made, the first part of which is based on the assumptions that the transition from the non-conducting to the conducting state of the arc is practically instantaneous, that when in the conducting state the arc obeys Ohm's law, and that the series inductance is very large. The second part is based on the last-named assumption and on the additional one that the product Px_m is constant. The principal results of this mathematical section are:

(1) The average counter E.M.F. P of an oscillating arc is given by

$$P = \frac{1}{2}\frac{v_0 + v_1}{1 + K}.$$

(2) If the impressed E.M.F. is constant, then to insure stability of the oscillations the mean value of the supply current x_m must satisfy the condition

$$x_m > \frac{1}{2} \cdot \frac{E}{R}.$$

(3) If the impressed E.M.F. is constant, then the efficiency cannot exceed 50 per cent.

(4) Stability can be obtained at a lower value of x_m than that given by the above inequality, and the 50 per cent. efficiency limit can be raised, by the use of an impressed E.M.F. which decreases rapidly with increase of current; the voltage changes, however, must follow the current changes almost instantaneously, if the method is to be effective.

(5) To obtain steady oscillations, the supply voltage (if constant) must satisfy the condition

$$E > \frac{v_0 + v_1}{1 + K}.$$

The importance of single-layer coils for the series inductance is discussed. Subsequent experiments with mercury vapor tubes in a Duddell circuit are then described, in which an impressed E.M.F. of 4,000 volts and a series inductance made up of fifty single-layer spirals were employed.

Again all efforts to obtain steady oscillations failed. The negative results can be accounted for by very high damping due to losses in the tube. It is shown that increasing the ratio of inductance to capacity in the oscillatory circuit is beneficial. This ratio, however, could not be increased sufficiently to give steady oscillations with the voltage available.

It is found that two parallel discharges in the same tube, each from its own cathode to a common anode, are much more unstable than a single discharge. The development from this discovery of a double cathode oscillator is described. By preliminary warming of the tube and by increasing the inductance-to-capacity ratio to 10^7, or more, steady oscillations are finally obtained by this method. The frequency of the oscillations did not differ by more than 2 per cent. from the natural frequency of the oscillatory circuit.

The writer wishes to thank Professor M. I. Pupin for his invaluable advice throughout the course of the experiments, and Professor G. B. Pegram for his help and kindly interest. He also wishes to thank Professor Wills for aid in revising the manuscript, Professor Arendt and Mr. Hehre for the loan of lamp-banks, etc., and Mr. Cushman for his assistance in the construction of apparatus. The tubes for these experiments were constructed by the Cooper-Hewitt Company of New Jersey.

Hartley Research Laboratory in Electro-Mechanics,
Columbia University,
April, 1915.

THE EFFECT OF TEMPERATURE ON THE ABSORBED CHARGE AND ON THE CAPACITY[1] OF SOME ELECTRIC CONDENSERS.

BY ANTHONY ZELENY.

A STUDY of the effect of temperature on the absorbed charge in electric condensers may aid in the development of a more complete theory of this charge and lead to a better understanding of what specific factors produce the various observed differences in its magnitude and behavior. The experiments described in this paper were undertaken to obtain some additional data[2] for that purpose. The effect of temperature on the capacity of the condensers under observation was determined at the same time and these results are also given.

The term "*free charge capacity*"[3] will be used to designate the capacity as measured by the free charge augmented by that part of the absorbed charge which passes through the galvanometer during the small fraction of a second it is connected to the condenser. The time allowed for the discharge is not much greater than is necessary to insure the whole measurable part of the free charge to pass through the galvanometer. The term "*true capacity*"[3] will be used to designate the capacity as measured by the free charge alone. Its value is obtained by extrapolation from the "free charge" capacity values at several different known periods of discharge.

A diagrammatic sketch of the apparatus employed for the direct measurement of the absorbed charges is shown in Fig. 1. In the position of the keys shown, the condenser C is charged through a resistance R_1. When the key K_1 is tripped by the pendulum (not shown), the condenser is short-circuited through another resistance, R_2, usually from .01 to .03 second and until the key K_2 is tripped. Then the absorbed charge, as

[1] A preliminary paper was read at the Washington meeting of the Physical Society, Dec. 27–30, 1911; abstract, PHYS. REV., Vol. 34, p. 141, 1912.

[2] Bibliography of literature on absorbed charges appended to paper by von Schweidler, Ann. der Phys., Vol. 24, p. 766, 1907; Trouton and Russ, Phil. Mag., Vol. 13, p. 578, 1907; Malclès, Journ. de Phys., Vol. 8, p. 631, 1909; Grover, Bul. Bureau of Standards, Vol. 7, p. 495, 1911.

[3] A. Zeleny, PHYS. REV., Vol. 22, p. 65, 1906; A. Zeleny and A. P. Andrews, PHYS. REV., Vol. 27, p. 72, 1908; Curtis, Bul. Bureau of Standards, Vol. 6, p. 431, 1910.

fast as it is liberated, passes through the galvanometer until the circuit is opened at the key K_3. The interval during which the liberated part of the absorbed charge is allowed to flow can be varied by altering the distance between the keys K_2 and K_3. The values of the liberated absorbed charge were calculated from the throws on the assumption that the prolonged flow of the charge produced the same effect as though the whole quantity passed through the galvanometer before the coil moved an appreciable amount.

To measure the free charge capacity of the condensers, the electrical connection between the keys K_1 and K_2 in the apparatus of Fig. 1 was

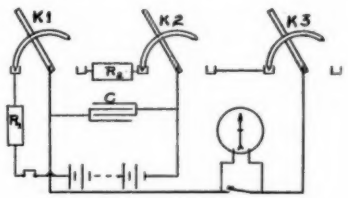

Fig. 1.

Apparatus for measuring the absorbed charge of an electric condenser, liberated in any given time. The keys are operated by means of a moving pendulum which is not shown.

removed, so as to avoid the short-circuiting. Then as soon as both of the keys K_1 and K_2 were tripped by the moving pendulum, the condenser began to discharge through the galvanometer and continued until the circuit was opened by the turning of the key K_3. In this manner each condenser was allowed to discharge through the galvanometer for .03 second, except in the case of condenser "*A* 1911" for which the period of discharge was 0.01 second.

A. Condensers within Metal Clamps.

§ 1. *Absorbed Charge of Paraffin-paper Condenser "A 1911."*

This condenser was made in ten sections, each of which was constructed from two sheets of tinfoil, 6.5 × 32 cm., and two sheets of rice paper 0.06 mm. in thickness. The sheets of rice paper were soaked in warm paraffin and assembled alternating with the sheets of tinfoil. The assembled sheets were then folded, with folds about 5 cm. in length, to form a "rolled" condenser. This, after pressing between flat surfaces, completed the construction which was the same for each of the ten sections. The ten sections were pressed together, connected in parallel, and placed in an iron box, which was then filled with melted paraffin. The cover of the box was at right angles to the plane of the sheets; and therefore the condenser with the paraffin surrounding it may be con-

sidered as clamped between metal plates. The determined values of the "free charge" capacity and of the "true" capacity, at different temperatures, are given in Table I.

TABLE I.

Condenser "A 1911."

Temperature.	Free Charge Capacity (.01").	True Capacity.	Difference.
21.°76 C.	.05368 mcf.	.05340 mcf.	.00028 mcf.
8. 65	.05486	.05469	17
0. 00	.05507	.05498	9
− 6. 09	.05524	.05519	5
−19. 5	.05524	.05521	3

The differences in the magnitude of the two kinds of capacity are plotted in Fig. 2. It is believed that this curve gives approximately the amounts of the absorbed charge, per volt, liberated at the different temperatures during the first .01 second of the discharge. The charging electromotive force was 42 volts; the resistance of the galvanometer,

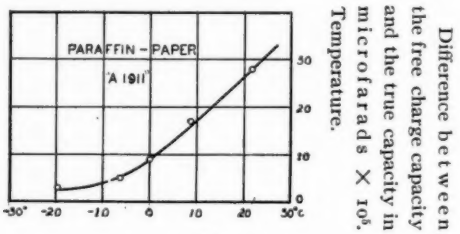

Fig. 2.

The absorbed charge of paraffin-paper condenser "*A* 1911," liberated during the first .01 second including the time during which the free electricity was discharging.

130 ohms; and the period of the galvanometer throw, 3.0 seconds. The resistance of the condenser, measured by the leakage method, varied from 58,000 megohms at 21.°64 to 302,000 megohms at − 19.°5 C.

The values of the absorbed charge for various longer periods of discharge from the same condenser were determined by the direct method described above and illustrated in Fig. 1, and are plotted in Fig. 3. The electromotive force of the charging battery was 124 volts, and each of the resistances R_1 and R_2, 130 ohms. The numerals placed beside each curve give, in seconds, the time allowed for the absorbed charge to flow through the galvanometer after the opening of the short-circuit. The absorbed charge liberated during the .01 second that the condenser was short-circuited, did not pass through the galvanometer.

§ 2. *Absorbed Charge of Some Air-mica and of Some Paraffin-paper Condensers.*

This set of condensers consisted of two air-mica and two paraffin-paper condensers, one of each kind having its dielectric untreated, and the other one, having its dielectric baked in an oven for twenty-four hours at a temperature of about 120° C. The tinfoil used was also similarly heated. The mica was assembled with the tinfoil without the use of paraffin, while for the paraffin-paper condensers the paper was dipped into heated paraffin. Each condenser consisted of 5 sheets of tinfoil, 6 × 10 cm., separated by the dielectric, and was clamped between metal plates by brass screws. After the condensers were clamped,

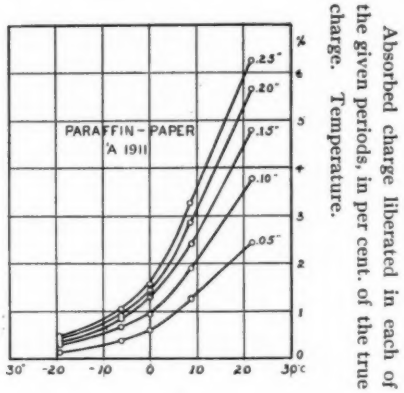

Fig. 3.

The absorbed charge of paraffin-paper condenser "*A* 1911," liberated during the designated periods immediately following the short-circuiting of the condenser for .01 second.

the edges were sealed with paraffin to prevent any possible condensation of moisture upon the condenser.

The apparatus of the previous experiment was employed, except that a Leeds and Northrup high sensitivity galvanometer was substituted for the type P instrument. The period of the ballistic throw was 4.5 seconds, and the resistance of the galvanometer coil, 650 ohms.

The condensers, after being charged one minute, were short-circuited for .03 second through a resistance of 1,000 ohms; and the liberated absorbed charge was allowed to flow through the galvanometer during the whole period of its throw. The data concerning the condensers are tabulated in Table II.

The quantities of the absorbed charge, per volt, liberated during the throw of the galvanometer coil are plotted in Fig. 4. They are assumed to be proportional to the voltage.

TABLE II.

Label.	Dielectric.	Treatment of Dielectric.	Charging Voltage.	Remarks.
I	Air-mica	Baked	78.2	Thickness of mica, .003 mm.
3 II	Air-mica	Untreated	1.4–2.9	Relative humidity of room at time of construction, 72 per cent.
3 VI	Paraffin-paper	Baked	27.7–51	Paraffin boiled in vacuum immediately preceding its use.
33 V	Paraffin-paper	Untreated	4.4–23	Paraffin only just melted.

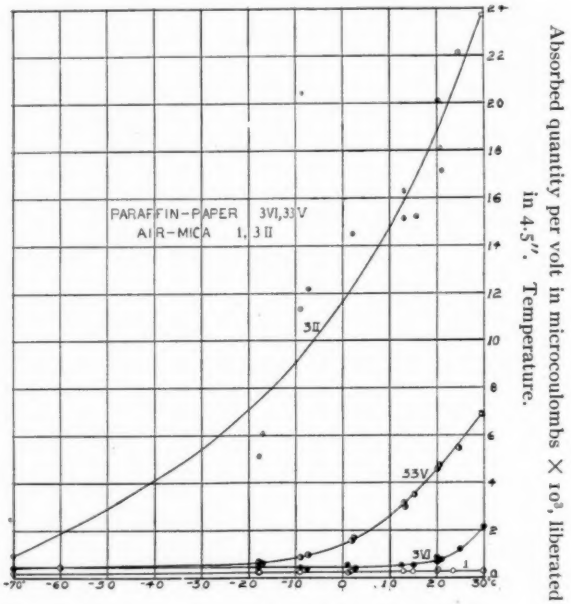

Fig. 4.

The absorbed charge of two air-mica and two paraffin-paper condensers, liberated during the whole period (4.5 seconds) of the ballistic throw immediately following the short-circuiting of the condenser for .03 second.

The same quantities are plotted in Fig. 5, with the ordinates enlarged 10 times.

The same results are again plotted in Fig. 6, but in per cent. of the free charge, the data being taken from the curves of Figs. 4 and 5 and from the corresponding capacities obtained from the curves of Fig. 8.

§ 3. *Capacity of the Clamped Condensers Employed in the Experiments of § 1 and § 2.*

The free charge capacity of the clamped condensers of § 1 and § 2 was determined at the various temperatures employed in the observations

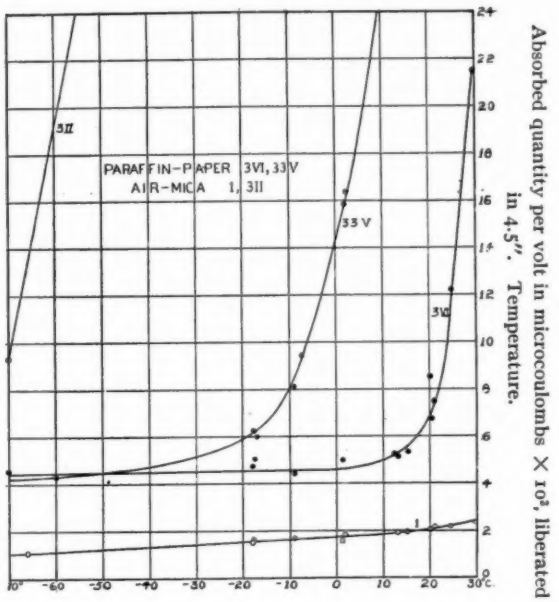
PARAFFIN-PAPER 3VI, 33V
AIR-MICA 1, 3II
3II
33V
3VI
1
Absorbed quantity per volt in microcoulombs × 10², liberated in 4.5". Temperature.
-70° -60 -50 -40 -30 -20 -10 0 10 20 30°C.

Fig. 5.

The absorbed charge plotted from the same data as the curves of Fig. 4, except that the scale of the ordinates was increased by the factor 10.

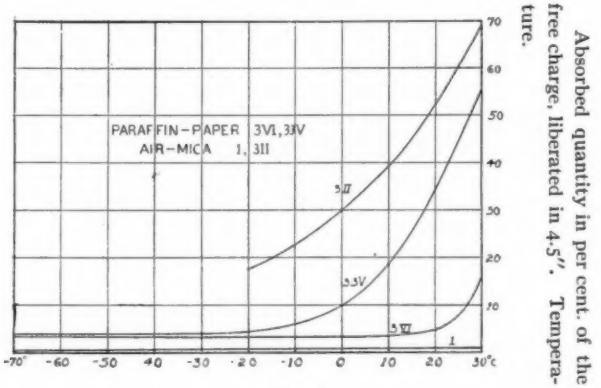
PARAFFIN-PAPER 3VI, 33V
AIR-MICA 1, 3II
3II
33V
3VI
1
Absorbed quantity in per cent. of the free charge, liberated in 4.5". Temperature.
-70° -60 -50 -40 -30 -20 -10 0 10 20 30°C.

Fig. 6.

The absorbed charge shown in Figs. 4 and 5 plotted in per cent. of the free charge given in Fig. 8.

on the absorbed charge. The results obtained are plotted in Figs. 7 and 8.

The ordinates for condenser "*A* 1911," in Fig. 7, are plotted on a larger scale than those for the other condensers, because there is a greater consistency in the position of the observed points. The observations

on this condenser were taken in order from the highest to the lowest temperature; while in the case of the condensers of Fig. 8 some of the observed points were not taken in consecutive order, or were repetitions

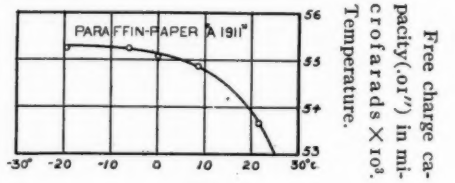

Fig. 7.

The free charge capacity (.01 second) of the clamped paraffin-paper condenser "*A* 1911" whose absorbed charge is plotted in Figs. 2 and 3.

after observations at other temperatures. Since the irregularity in the position of the plotted points is much greater than the probable observa-

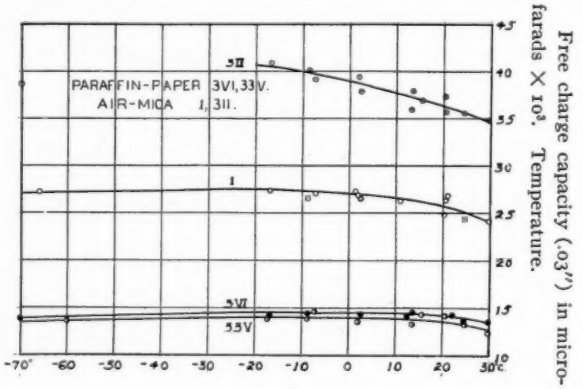

Fig. 8.

The free charge capacity (.03 second) of the two clamped air-mica and the two clamped paraffin-paper condensers whose absorbed charge is plotted in Figs. 4, 5, and 6.

tional error, permanent changes in the capacity appear to have taken place with a change in temperature.

The mean temperature coefficients of the free charge capacity, between 0° and 20° C., for the clamped condensers as obtained from the above curves, are given in Table III.

TABLE III.

Temperature Coefficients of the Clamped Condensers.

Label.	Dielectric.	Treatment of Dielectric.	Change in Capacity per Unit Capacity per Degree, between 0° and 20° C.
3 VI	Paraffin-paper	Baked	-1.03×10^{-3}
33 V	Paraffin-paper	Untreated	-1.61
"*A* 1911"	Paraffin-paper	Untreated	-1.13
1	Air-mica	Baked	-2.40
3 II	Air-mica	Untreated	-3.33

B. UNCLAMPED CONDENSERS.

§ 4. *Absorbed Charge of Some Paraffin-paper and of Some Ozokerite-paper Condensers.*

This set of condensers was constructed to test especially the effect on the absorbed charge produced by varying amounts of moisture within the same dielectric. The dielectric was rice paper, saturated in a part of the condensers with "paraffin," and in the others, with "ozokerite." All of the condensers were constructed of approximately the same size and in the same manner as one of the sections of condenser "*A* 1911," described in § 1. The condensers, unclamped, were placed into metal boxes which in this case were not filled with paraffin. The boxes, before sealing, contained very dry air, to prevent, later, a precipitation of moisture at low temperatures.

The observations on this set of condensers were taken previous to those of § 2, but in a similar manner; the apparatus being the same with the exception of the galvanometer which was of the same type but having

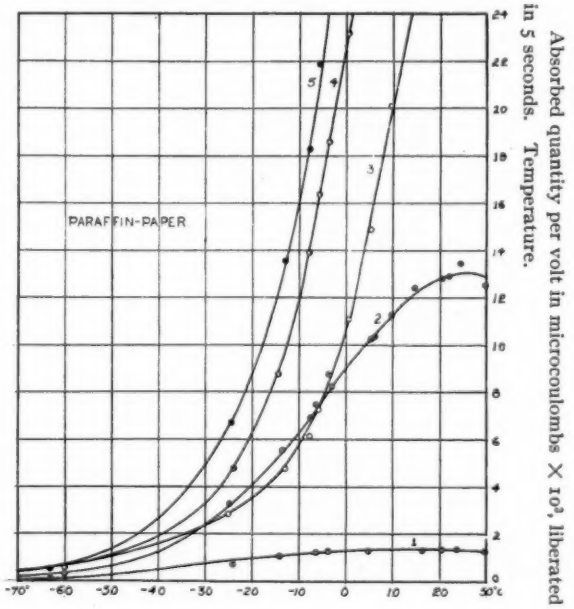

Fig. 9.

The absorbed charge of 5 paraffin-paper condensers containing different amounts of moisture in the dielectric, liberated during the whole period (5 seconds) of the ballistic throw, immediately following the short-circuiting of the condenser for .03 second.

a resistance of 670 ohms and the period of throw, 5 seconds. The condensers are labeled with numerals to indicate increasing amounts of

moisture in the dielectric. Through an accident, some doubt is cast on the correctness of this labeling, but it is believed to be correct. The treatment of the paraffin and of the ozokerite in the differently labeled condensers was as follows:

1. Boiling under slightly reduced pressure.
2. Blowing dry air through the melted wax.
3. Untreated.
4. Blowing moist air from the lungs through the melted wax.
5. Forcing steam through the melted wax.

The absorbed charges obtained from the paraffin-paper condensers are plotted in Fig. 9, and those from the ozokerite-paper condensers, in Fig. 10.

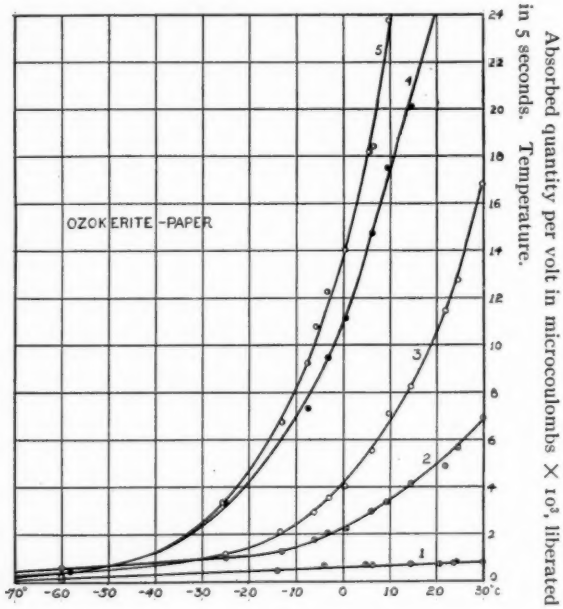
OZOKERITE-PAPER
Absorbed quantity per volt in microcoulombs × 10³, liberated in 5 seconds. Temperature.
-70° -60 -50 -40 -30 -20 -10 0 10 20 30°C
0 2 4 6 8 10 12 14 16 18 20 22 24
1
2
3
4
5

Fig. 10.

The absorbed charge of 5 ozokerite-paper condensers containing different amounts of moisture in the dielectric, liberated during the whole period (5 seconds) of the ballistic throw, immediately following the short-circuiting of the condenser for .03 second.

§ 5. *Capacity of the Unclamped Condensers Employed in the Experiments of § 4.*

The free charge capacities for the paraffin-paper condensers employed in the experiments of § 4, are plotted in Fig. 11, and for the ozokerite-paper condensers, in Fig. 12.

The mean temperature coefficients of the free charge capacity, between 0° and 20° C., for the unclamped condensers as obtained from the curves of Figs. 11 and 12, are given in Table IV.

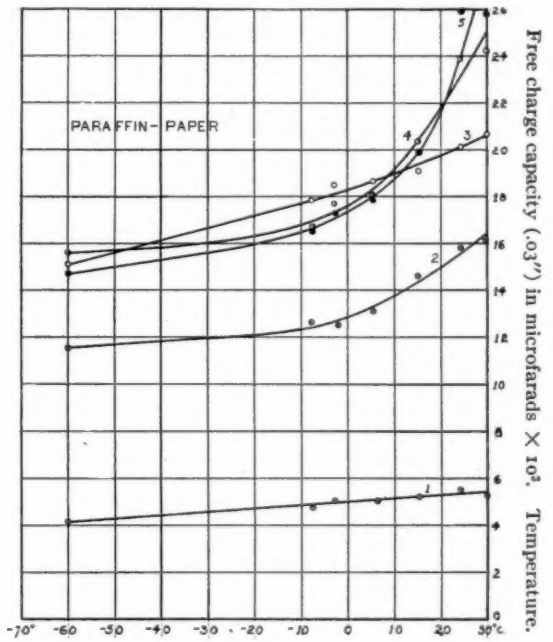

Fig. 11.

The free charge capacity (.03 second) of the 5 unclamped paraffin-paper condensers whose absorbed charge is plotted in Fig. 9.

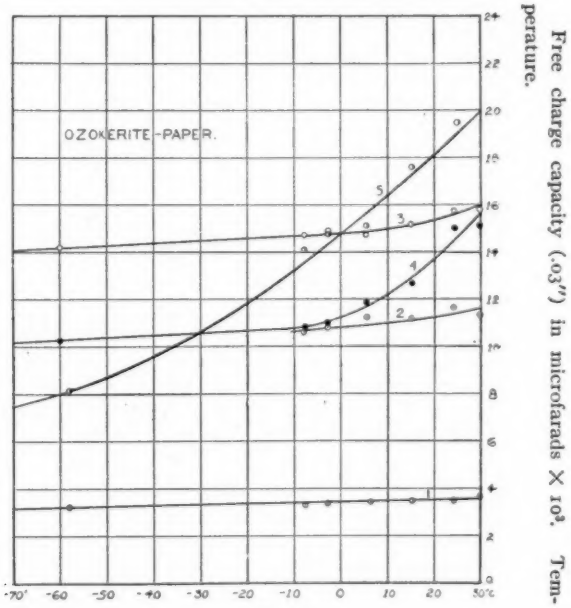

Fig. 12.

The free charge capacity (.03 second) of the 5 unclamped ozokerite-paper condensers whose absorbed charge is plotted in Fig. 10.

TABLE IV.

Temperature Coefficients of the Unclamped Condensers.

Label.	Treatment of Dielectric.	Change in Capacity per Unit Capacity per Degree, Between 0° and 20° C.	
		Paraffin-Paper.	Ozokerite-Paper.
1.	Partial vacuum.....	2.5×10^{-3}	1.5×10^{-3}
2.	Dry air.....	8.4	2.0
3.	Untreated.....	4.1	1.9
4.	Moist air.....	11.9	11.0
5.	Steam.....	12.2	11.3

SUMMARY.

The observations given in this paper lead to the following statements in regard to the condensers tested:

1. An increase in the amount of moisture within the dielectric produced a great increase in the magnitude of the absorbed charge at the higher temperatures, and had comparatively little effect at very low temperatures.

2. There is a narrow range of temperature within which, for any particular scale selected for plotting, the absorbed charge curve has a marked change in direction. This has but little more significance than that of indicating where, with the degree of precision employed in the measurement, the change in the magnitude of the liberated absorbed charge becomes considerable. Although the magnitude of the succeeding increments increases with temperature the rate of the increase appears to follow no general law.

3. The greater the amount of moisture in the dielectric, the lower is the temperature at which the absorbed charge, for any particular degree of precision employed, begins to increase rapidly with temperature. This may be explained in the following manner. The greater the amount of moisture in the dielectric, the greater is the number of particles taking part in producing the absorbed charge; and consequently the absorbed charge is greater at any given temperature, and the temperature at which it becomes large enough to have significant magnitude is lower.

4. In those condensers in which the greater part of the absorbed charge is due to moisture, the absorbed charge curve, in general, shows no sudden change in the magnitude of the absorbed charge near the freezing point of water.

5. Clamping of the condensers appears to have no interpretable influence on the absorbed charge.

6. A mica condenser may have a larger absorbed charge than a poor paper condenser.

7. The presence of moisture in the dielectric apparently increases the free charge capacity of a condenser, as well as the magnitude of the absorbed charge.

8. In some of the condensers there is a narrow range of temperature within which the free charge capacity curve, on any chosen scale, has a marked change in direction similar to that of the absorbed charge curve.

9. The capacity curve does not have the portion containing the marked change in direction at the same temperature as the temperature of the similar portion in the absorbed charge curve of the same condensers, when plotted on the same scale.

10. In all the clamped condensers studied, the free charge capacity had a negative temperature coefficient, and in all the unclamped condensers, a positive temperature coefficient. It is of interest to compare this with the observations on commercial condensers taken by Curtis[1] and by Grover.[2]

The graduate school of the University of Minnesota provided assistants from its research fund, who aided in the construction of the condensers and in taking many of the observations.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
July 21, 1915.

[1] Bul. Bureau of Standards, Vol. 6, p. 447, 1910.

[2] Bul. Bureau of Standards, Vol. 7, p. 495, 1911.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NEW YORK MEETING, OCTOBER 30, 1915.

THE seventy-ninth meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York, on Saturday, October 30, 1915. Two sessions were held, at 9.30 A.M. and 2 P.M., President Merritt presiding.

The following twenty-eight papers were presented:

A Modified Form of the Moving Lamp Photometer. C. C. TROWBRIDGE AND W. B. TRUESDELL.

Demonstration of a Model Visibly Representing Gas Molecules. E. F. NORTHRUP.

The Fields of Force Surrounding Atoms. ALBERT C. CREHORE.

A New Method of Analyzing the Motion of a Vibrating String. W. S. FRANKLIN.

The Theory of the Flicker Photometer. II. Unsymmetrical Conditions. HERBERT E. IVES AND E. F. KINGSBURY.

The Velocity of Polymorphic Changes Under Pressure. P. W. BRIDGMAN.

Axial Interference Figures from Brookite with Light of Different Wavelengths. JOHN B. TAYLOR.

A New Line of Argument for the Rigorous Establishment of the Idea of Entropy as a Measurable Quantity. W. S. FRANKLIN AND BARRY MCNUTT.

A New Type of String Galvanometer. HORATIO B. WILLIAMS.

Some Data on the Absorbed Charge in Electric Condensers. ANTHONY ZELENY.

The Temperature Coefficient of Contact Potential. KARL T. COMPTON.

Equipartition of Energy and Radiation Theory. (By title.) W. F. G. SWANN.

Planck's Radiation Formula Deduced from Hypotheses Suggested by X-ray Phenomena. WILLIAM DUANE.

The Relations of the Photo Potentials Assumed by Different Metals When Stimulated by Light of a Given Frequency. A. E. HENNINGS AND W. H. KADESCH.

Radiation from Tungsten Filaments and the Mechanical Equivalent of Heat. IRVING LANGMUIR.

The Construction of Compound Molecules by Means of the Fields of Force, Especially the Systems of Growth of the Organic Compounds of Hydrogen and Carbon. ALBERT C. CREHORE.

Some New Measurements on the Thermometric Movement of Tree Branches. C. C. TROWBRIDGE.

An Interesting Phenomenon of Wave Motion. W. S. FRANKLIN.

The Value of h determined Photoelectrically from the Ordinary Metals. W. H. KADESCH AND A. E. HENNINGS.

Heat Conductivity of Tungsten at High Temperatures and the Wiedemann-Franz Relation. IRVING LANGMUIR.

Atomic Groups in Solids as Indicated by their Thermal Conductivities. ARTHUR H. COMPTON.

A Relation between the Melting Points of Substances and their Heats of Formation. ARTHUR H. COMPTON.

Negative Resistance. ALBERT W. HULL.

Note on an Atomic Model giving a Spectrum Series with Constant Frequency Intervals. ERNEST MERRITT.

The Resistance and Thermo-Electric Power of the Oxides of Iron. C. C. BIDWELL.

A Turbine Type of Molecular Pump. GEO. B. PEGRAM.

Einstein's Photo-electric Equation and Contact E.M.F. R. A. MILLIKAN.

A New Derivation of Planck's Radiation Formula. LEIGH PAGE.

The attendance at each session exceeded one hundred. One hundred and seventeen registered. The next meetings of the Society are at Chicago, November 26 and 27, and at Columbus, December 28–30.

A. D. COLE,
Secretary.

A NEW METHOD OF DETERMINING THE AMPLITUDE OF SOUND WAVES IN AIR.[1]

BY E. P. LEWIS AND L. P. FARRIS.

ESTIMATES of the amplitude of sound vibrations have been made from pressure measurements by Toepler and Boltzmann and Kundt, from energy relations by Rayleigh, and from the vibrations of telephone diaphragms by Shaw. Mach has observed the vibrations of columns of vapor due to sound waves, and Boys set luminous particles ascending from a flame in vibration in the same manner. We have, however, found no discussion of the relations between the amplitudes of vibration of the objects used and those of the sound waves.

It occurred to one of us that the amplitude of the sound waves might be determined from dynamical relations from the observed displacement of illuminated microscopic particles set into forced vibrations by the waves. Preliminary observations showed that suspended particles of ammonium

[1] Abstract of a paper presented at the meeting of the American Physical Society, August 2–7, 1915.

chloride, lycopodium powder, etc., actually participate in the vibrations of the sound waves passing by them. The particles were dropped through a tall chimney-like box of small cross-section with openings on three sides at the same level. Through one a beam of sunlight passed and illuminated the particles. Into the box from the opposite side projected the small end of a funnel, the large end of which was covered with a paper diaphragm. Through the third opening was placed the objective of a microscope which included in its field of view the region adjacent to the open end of the funnel. Near the diaphragm was the open end of an organ pipe of frequency 230. When this sounded the diaphragm was set into vibration, and the sound waves in the funnel, with magnified amplitude at the small end, set the particles in vibration as they fell past the objective of the microscope. Lycopodium powder was the most satisfactory substance tried, as the particles were large enough to fall steadily with no appreciable brownian movement. When the organ pipe was silent the paths of the particles appeared like bright straight streaks against a dark background; when it sounded these lines broke up into well-defined sine curves. Even with a magnifying power of 90 diameters the amplitude of these curves was small, but it is possible to photograph them and to make accurate measurements.

The relation between the amplitude of the particles and that of the sound waves is a simple one. The displacement in the sound wave is

$$x = A \sin \omega t. \tag{1}$$

That of the particle is of the same period but of different amplitude and phase:

$$u = a \sin (\omega t - \varphi). \tag{2}$$

At a given instant the force acting on the particle is proportional to the relative velocity between it and the surrounding medium. If m is the mass of the particle,

$$m \frac{d^2u}{dt^2} = - ma\omega^2 \sin (\omega t - \varphi) = k \left(\frac{dx}{dt} - \frac{du}{dt} \right). \tag{3}$$

Differentiate (1) and (2) with respect to the time, substitute in (3), expand, and equate coefficients of $\sin\omega t$ and $\cos\omega t$. The solution is found to be

$$\tan \varphi = \frac{m\omega}{k}, \tag{4}$$

$$A = \frac{a}{\cos \varphi}. \tag{5}$$

If the particle while vibrating in a horizontal direction falls with a constant velocity v,

$$mg = kv. \tag{6}$$

Eliminating k, (4) becomes

$$\tan \varphi = \frac{\omega v}{g}. \tag{7}$$

A photograph of the sinusoidal curve representing the path of the particle is a chronographic record from which the velocity and the amplitude of the particle may be determined if the frequency of the sound waves and the magnifying power of the system are known, and from these data A may be calculated.

Rough estimates made with a micrometer eyepiece gave the following results, which at least indicate the order of magnitude of the amplitude.

$$n = 230,$$

$$a = 0.0006 \text{ cm.},$$

$$v = 7 \text{ cm./sec.},$$

$$\tan \varphi = 10.35, \quad \varphi = 84°.5,$$

$$A = \frac{a}{\cos \varphi} = 0.0063 \text{ cm.}$$

Further experiments are in progress for the purpose of obtaining quantitative results and determining whether it is legitimate to assume that k is constant for accelerated motion.

PHYSICAL LABORATORY,
UNIVERSITY OF CALIFORNIA.

A QUANTITATIVE DETERMINATION OF THE EARTH'S PENETRATING RADIATION.[1]

BY CHARLES HENRY KUNSMAN.

THE determination is based upon the assumption that the discharge of an electroscope is due to a leakage across the insulation support and to ionization in the vicinity of the leaf. The source of this radiation is shown to be due principally to the radioactive material in the earth and the atmosphere.

A Spindler and Hoyer aluminum leaf electroscope, with adjustable ionization chambers was employed. By varying the size of the ionization chamber, the ionization effects were separated from the leak across the insulation support. The number of ions formed per c.c. per second, due to the penetrating radiation, may be represented by

$$n = \frac{C_1 \delta E_1 - C_2 \delta E_2}{(V_1 - V_2) \times e},$$

where C_1 and C_2 are the respective capacities, δE_1 and δE_2 the corresponding drop in potential with the respective volumes of the ionization chambers V_1 and V_2, and e the unit electric charge.

The mean value of n as observed in the basement of South Hall was 8.68, and on the Pacific Ocean from San Francisco to Los Angeles 4.15. A lead screen reduced the ionization on land to about one half its original value. It

[1] Abstract of a paper presented at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

was also shown that the ionization due to the radioactive material in the atmosphere was not far from .5 ion per c.c. per second. It is interesting to note that the ionization on the ocean and on the land, with the instrument screened, seems to approach a minimum which is about 4 ions.

PHYSICAL LABORATORY,
UNIVERSITY OF CALIFORNIA.

THE ULTRA-VIOLET SPECTRA OF KRYPTON AND XENON.[1]

BY E. P. LEWIS.

THE most complete study of the spectra of these gases has been made by Baly,[2] but his published results do not extend beyond about λ 2415. Photographs of the spectra of the discharge in quartz vacuum tubes furnished by Hilger showed many bright lines extending to about λ 2150, the limit of transmission of the thick quartz system used. The wave-lengths of several hundred new lines have been determined.

Many spectra given by the discharge with condenser and spark gap in the circuit are greatly modified by the addition of self-induction. This is notably true in the case of krypton. A very small inductance will greatly reduce the intensity of the discharge, but by properly increasing the time of exposure it will be found that the intensity of a small number of lines is about the same as in the spectra taken without inductance; a much larger number of strong lines appear to be completely suppressed, while a number of other lines remain, but with greatly reduced intensity.

Baly found that the spectrum of the simple discharge in krypton (the first spectrum) is entirely different from that of the condensed discharge (the second spectrum), and terminated about λ 3500. It is found that the spectrum of the simple discharge extends much further than this, and that, besides the lines of the first spectrum, it includes most of the strong lines which persist with self-induction. This effect is not so pronounced in the case of xenon, although some lines are common to the two spectra. A number of numerical relationships between krypton lines not suppressed by self-induction have been found.

PHYSICAL LABORATORY,
UNIVERSITY OF CALIFORNIA.

TESTS OF THERMO-ELECTRIC FORMULÆ BASED ON BISMUTH AND BISMUTH-TIN ALLOYS.[1]

BY A. E. CASWELL.

THIS paper presents experimental data on the Peltier E.M.F. and thermo-electric power of pure bismuth and of alloys of bismuth and tin containing small percentages of tin.

[1] Abstract of a paper presented at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

[2] Phil. Trans., *A* 202, 183, 1903.

It is shown that if the mean free path of the electrons in an alloy is practically the same as in the pure bismuth, and if the electrons which take part in electrical and thermal conductivity are identical with those which take part in thermo-electric phenomena, we have a means of testing the thermo-electric formulae deduced from the electron theory by different authors using different methods.

According to Drude the Peltier E.M.F. should satisfy the equation

$$P = \frac{4\alpha T}{3e} \log \frac{n_1}{n_2},$$

where α is a universal constant, T is the absolute temperature, e is the charge on an electron, and n_1 and n_2 are the numbers of electrons in unit volume of the two metals considered. This formula is equivalent to

$$P = \frac{2RT}{e} \log \frac{n_1}{n_2},$$

where R is the gas constant for a gram-molecule of a gas, and e is the electrochemical equivalent for hydrogen. The latter equation is identical with that deduced by J. J. Thomson and O. W. Richardson if the factor 2 is omitted. The fundamental assumptions in each case have been different. Similarly, in accordance with Lord Kelvin's equation $P = QT$, where Q is the thermo-electric power, we have

$$Q = \frac{2R}{e} \log \frac{n_1}{n_2}.$$

Using the experimental data of Schulze[1] on electrical and thermal conductivities of bismuth and its alloys together with the data given in this paper, it is found that Drude's formulæ are satisfied, but Thomson's are not. The character of the agreement between the data obtained from different sources indicates that the assumptions made in this paper are justified. It is believed that this work definitely links the phenomena of thermo-electricity to those of conductivity in metals.

Per Cent. Tin.	Peltier Effect.	Thermo-electric Power.	Per Cent. Tin.	Electric Conductivity.	Thermal Conductivity.
0	1.000	1.000	0	1.000	1.000
—	—	—	0.34	0.717	0.741
—	—	—	0.71	0.610	0.667
3.75	0.550	0.552	2.15	0.543	0.578
6.36	0.542	0.528	7.43	0.588	0.662
9.93	0.555	0.543	—	—	—

The following table gives the values of the ratio n_1/n_2 computed from various sources, the subscript 1 referring to the alloy and the subscript 2 to pure

[1] Ann. der Phys., 9, pp. 555–589, 1902.

bismuth. The first and fourth columns give the percentages by weight of tin in the alloy.

The work is being continued and its scope extended.

PHYSICAL LABORATORY,
UNIVERSITY OF OREGON.

AN EXPERIMENTAL VERIFICATION OF THE LAW OF VARIATION OF MASS WITH VELOCITY FOR CATHODE RAYS.[1]

BY LLOYD T. JONES.

THE method used in the present verification of the law of variation of mass with velocity is a modification of the usual electrostatic and magnetic deflection photographic method. The method was first used by the writer in a preliminary work for the determination of e/m for cathode rays.[2] With a few minor changes which greatly increased the accuracy of the values obtained it has been applied to the investigation of the variation of the mass of the electron.

The method has the distinct advantage of employing a non-homogeneous cathode beam. The traces on the photographic plate record the paths of electrons of all velocities present. A transformer affording 150,000 volts was used without need of a rectifier or apparatus to produce a uniform discharge potential.

A number of photographs were taken and, due to the great range of velocities present, a verification of the law was obtained from *each* photograph. The results verify the Lorentz-Einstein formula. The range of velocities obtainable with the apparatus was from 5×10^9 cm./sec. to 1.5×10^{10} cm./sec.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
June, 1915.

MAGNETIC FIELDS PRODUCED BY ROTATING SOLID CONDUCTORS IN A MAGNETIC FIELD.[1]

BY S. R. WILLIAMS.

IF a solid aluminum cylinder is rotated in a magnetic field so that the axis of rotation is normal to the field in which it is rotated, a magnetic field is set up normal both to the axis of rotation and to the direction of the applied magnetic field. In this paper a detailed study is made of the magnetic field which is thus established by rotation and some conclusions are drawn as to its bearing upon the problem of magnetization by rotation.

PHYSICAL LABORATORY,
OBERLIN COLLEGE, OBERLIN, OHIO.

[1] Abstract of a paper presented at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

[2] Jones, PHYS. REV., N. S., III., p. 317, 1914.

The Heat Losses from Incandescent Filaments in Air.[1]

By L. W. Hartman.

FILAMENTS of platinum sponge were mounted in front of an electric furnace, which was subject to temperature control, and heated by means of a current from a storage battery. Through a telescope one could observe these wires against a background of magnesium oxide mounted at the center of the furnace and as they were brought to incandescence a point was finally attained where the wires and the furnace had the same appearance. At this point the wires were assumed to have the same temperature as the interior of the furnace. The power supplied to the wires was then measured and the temperature of the furnace was determined with the Wanner pyrometer. Assuming that these filaments were black bodies and that they radiated energy in accordance with the Stefan-Boltzmann law, the energy loss due to convection and conduction was determined. From these data and observations curves were plotted to show the variation of the energy radiated, the energy supplied, and the energy lost with change of temperature.

In order to separate the conduction and convection losses the effect of introducing the filaments in a vacuum was tried. Curves were then plotted from the data thus obtained, corrected for the absorption of the glass vessel, to show how these losses varied with rise of temperature. This procedure was carried out with both Coblentz's and Gerlach's determination of the radiation constant in order to see which constant, in the light of these data, was the most probable value. As a whole the results seemed to favor the lower value of the radiation constant.

From the tabulated data and the curves it appears that the equation connecting the conduction and convection losses, L, may be expressed thus:

$$L = kT,$$

where T is the absolute temperature and k is a constant for the given wire.

By applying Lorenz's equation to the energy supplied in the case of the first wire one can compute by the method of least squares the two coefficients required in this equation. Adopting Tereschin's assumption that for the different sized wires the radiation coefficient of this equation varies directly with the surface of the wires, the coefficients for the other wires were determined. Computing, with the aid of these coefficients, from Lorenz's equation the energy supplied to each wire, it was found that the observed curves coincide with great exactness with the curves obtained from the computation with Lorenz's equation. It was further found that the curves showing the variation of the energy lost by conduction and convection with temperatures between 900–1700° Absolute coincided more closely with the curve obtained from the equation

$$L = kT$$

[1] Abstract of a paper read at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

than from the curve plotted from the values of losses obtained by using Tereschin's method.

UNIVERSITY OF NEVADA.

ULTRA-VIOLET ABSORPTION SPECTRA.[1]

BY R. L. SEBASTIAN.

THE purpose of this investigation was to obtain the ultra-violet absorption spectra of benzene and some of its monosubstitution products, using a new source of illumination. The usual methods of illumination, the cadmium or aluminum spark under water, have always been a source of more or less trouble inasmuch as the water soon becomes turbid and must be changed often. Then, too, the spectra given by these metals are not perfectly continuous.

E. P. Lewis proposed the use of the simple discharge of the hydrogen tube as a source of illumination for absorption spectra in the ultra-violet,[2] which has been shown to give a continuous spectrum in that region. Schniederjost[3] and Frederichs[4] had observed that hydrogen gave a continuous spectrum in the ultra-violet, and the latter used it in trying to obtain ultra-violet absorption spectra using a tube of capillary size which required exposures from twelve to twenty-four hours to obtain a satisfactory plate. By the use of the tube designed by Professor Lewis satisfactory exposures may be obtained in from fifteen to forty-five minutes depending upon the width of the slit and the absorption power of the medium. The tube is a large one, used end on, about thirty centimeters in length and about twelve millimeters in diameter, and has a quartz plate sealed in the end. A pressure of from four to seven millimeters of hydrogen is used. It does not make any material difference if the hydrogen is not completely purified as a little air or water vapor has no noticeable effect upon the spectra, which show perfectly continuous as far as the thick quartz system transmits, *i. e.*, to about λ 2100. The large two-prism quartz spectrograph previously described[5] has a dispersion in the ultra-violet region greater than that given by the first order of a 5-inch grating.

While this work was in progress two articles from Kayser's Laboratory at Bonn have appeared: "Die ultraviolette Absorption einiger Monoderivate des Benzols,"[6] by Christine Strasser; "Zur Strucktur der ultravioletten Absorptionsspektren des Benzols und seiner Monoderivate,"[7] by Rudolph Witte. Both writers have found fairly constant differences between the groups

[1] Abstract of a paper presented at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

[2] Science, N. S., Vol. XLI., No. 1069, pp. 947–48, June 25, 1915.

[3] Z. f. wiss. Photo., p. 265, 1904.

[4] Bonn Diss., 1905.

[5] Astrophysical Journal, Vol. 23–5 (June, 1906).

[6] Z. für wiss. Photographie, Photophysik und Photochemie, Vol. XIV., No. 9, pp. 281–311 (1915).

[7] Z. für wiss. Photographie, Photophysik und Photochemie, Vol. XIV., Nos. 11 and 12, pp. 348–391 (1915).

of benzene, and also between the groups of the mono-substitution products, showing that the benzene ring is not changed by the addition of a mono-group.

It may be stated, however, that with the better source of illumination and greater dispersion of the spectrograph many new absorption bands have been found which are not mentioned in either of the above papers. It will be interesting to see if the relationships they claim to have found will hold when these new bands are taken into account.

Physical Laboratory,
University of California.

The Oxide Resistance Thermometer.[1]

By S. L. Brown.

THE rapid variation of resistance of the metallic oxides with temperature has been utilized to measure temperature. The greatest obstacle in the use of the oxide resistance thermometer is the difficulty in making electrical connections to the oxides. Heretofore connections have been made by clamping wires against the ends of the oxide resistance-element. The author now makes electrical contact by brazing; that is, a small globule of the molten oxide and the molten ends of the wires are brought into contact and allowed to cool.

If good connections are made in this manner, a very satisfactory resistance thermometer can be had which will have a resistance of only 50 to 100 ohms at room temperature, but the resistance will be nearly doubled if its temperature is reduced to ice-point. In many cases, thermometers made in this manner will have many thousand ohms resistance. In all probability, poor connections to the wires accounts for this high resistance. The high resistance thermometers have variable resistances depending on the strength of the current sent through them. Heat produced at poor contact would decrease the resistance and hence the element would have lower resistance for higher currents. Such a high resistance element makes a very satisfactory and sensitive thermometer if used at constant current. Temperatures may be recorded in terms of the resistance which must be added in series with the thermometer in order to keep a constant current, which is furnished by a constant voltage supply. A temperature change of 20 degrees Centigrade may cause a resistance change of 15 or 20 thousand ohms in such a thermometer.

The University of Texas.

A New Form of Radiation Pyrometer.[1]

By S. L. Brown.

THE plan in this instrument is to replace the thermo-junction of the Fery radiation pyrometer with an oxide resistance-element. The radiation from the hot body whose temperature is to be measured is focused, by means

[1] Abstract of a paper read before the meeting of the American Physicial Society at San Francisco, Aug. 2–7, 1915.

of a concave mirror, on a very small and thin oxide element. This element is connected in series with a constant voltage battery and a high resistance millivoltmeter. The current, as indicated by the deflection of the millivoltmeter, is a measure of the temperature of the body whose radiation is focused on the high resistance oxide element.

The instrument may be adapted to any temperature interval by shunting the millivoltmeter with the proper resistance. For example, a temperature change from 1600 to 2100 degrees Fahrenheit may cause a deflection of the millivoltmeter from 50 to 125 with a particular shunt while this same district of the millivoltmeter scale may correspond to temperatures below 1600 degrees Fahrenheit when a higher resistance shunt is used.

THE LAW OF COHESION IN MERCURY.[1]

BY P. A. ROSS.

WHEN a monatomic liquid is warmed the heat is expended in three ways:

1. Increasing the kinetic energy of the molecules.
2. Expanding the liquid against external pressure.
3. Expanding the liquid against internal or cohesion pressure.

If the first is assumed to be equal to the specific heat of the substance as a gas at constant volume the value of the third may be computed since the sum of all three is equal to the specific heat of the liquid.

Using values for the specific heat and density of mercury at 5° intervals from 0° to 360° C. from Landolt-Börnstein Tabellen the relation of cohesion to molecular distance was found to follow very closely an inverse sixth power law.

The force per molecule was then computed and integrated for the removal of a gram of molecules to an infinite distance. This gave for the latent heat of vaporization values ranging from 41.1 calories (Milthaler's data) to 59 calories (Naccari's data) instead of the experimental value of 68 calories per gram.

LELAND STANFORD JUNIOR UNIVERSITY,
July 19, 1915.

THE APPLICATION OF PHYSICAL PRINCIPLES TO PROBLEMS SUGGESTED BY OCEANIC CIRCULATION AND TEMPERATURES.[1]

BY GEORGE F. MCEWEN.

MOST of our quantitative knowledge of the great ocean currents depends upon the difference between the true position of a ship and that determined from "dead-reckoning"; and upon observations of floating objects. This work has been supplemented by current meter observations in a few limited regions. Also the distribution of ocean temperatures, which is intimately connected with the motion of the water is an important though indirect source of information concerning ocean currents.

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

However the temperature data have been used only in a qualitative way for this purpose. This paper is a summary of some of the quantitative results I have obtained by attempting to calculate the velocities of both horizontal and vertical currents from the distribution of temperatures.

First, assume that the average flow of the water in any direction is negligible, and the depth exceeds 100 meters. Assume the rate of absorption of radiant heat by the water to be an exponential function of the depth, and the rate of loss to be proportional to this exponential function and to a linear function of the temperature. From the observed relation of the mean annual surface temperature and the annual range of the surface temperature to the latitude and also from the assumption that the amount of radiant energy penetrating the water surface is proportional to the solar radiation incident on a horizontal surface at that time and latitude, the normal temperature can be expressed as a function of the depth, latitude and time. The constants for this expression have been determined and the temperatures, computed for various positions, depths and times at which normal values would be expected, agreed well with the observed values.

The influence of a horizontal current was estimated by adding to the differential equation first used a term expressing the rate of gain of heat due to the difference in temperature between the water flowing into an element of volume and that flowing out. By assuming the velocity along stream-lines from various points along the west coast of North America to be proportional to the average wind velocity over them, the temperatures as modified by this surface drift were computed and found to agree well with the observed values. Moreover the relative velocity of the wind and current thus found were in good agreement with those determined in other localities by direct measurement.

The relatively low temperatures of the inshore water along the Pacific coast is regarded by many to be the result of the local up-welling of cold bottom water. Assuming a vertical current with a velocity proportional to that of the wind parallel to the coast as required by Ekman's theory of oceanic circulation serial temperatures corresponding to average monthly values off San Diego were computed and found to agree well with observations. Also some results computed from salinity observations confirmed the above conclusions regarding the circulation.

The author expects to continue the study of this problem and to pubish the methods and results in detail, as soon as possible.

Laboratory of the Scripps Institution for Biological Research.
La Jolla, California.

The Efficiency of Astronomical Spectrographs.[1]

By J. H. Moore.

TWO conditions of fundamental importance control the design of spectrographs for most work in astronomy, which in general do not have to

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

be met in the laboratory. First, celestial light sources are for the most part of feeble intensity; second, in order to avoid loss of light, the spectrograph must be mounted on a moving telescope and hence is subject to the varying component of gravity, as well as to the changing temperature of a well-ventilated dome.

The losses of light in the various parts of the combined telescope and prism spectrograph are considered and the broad principles indicated in accordance with which the various parts of the instrument must be arranged in order to give maximum light efficiency, when used for the study of the two different classes of celestial objects, stars and extended surfaces.

One of the most important factors in the design of stellar spectrographs which shall give spectra of maximum intensity is shown to be the proper balancing of the two conflicting elements, the transmission at the slit and the transmission of the prism train. A practical illustration is given of the relative efficiency of two spectrographs in actual use at Mount Hamilton in the determination of the radial velocities of gaseous nebulæ,—a one-prism spectrograph, and a three-prism with a camera of 1/3 the focal length of the former. For extended objects the three-prism is about 5 times as fast as the one-prism; for stellar objects, however, its efficiency is only about 50 per cent. greater than that of the one-prism.

The successful design of an astronomical spectrograph, which shall give maximum efficiency in any particular line of research, is a bundle of fortunate compromises between the various conflicting elements of light transmission in the optical system and the construction and arrangement of its component parts so that they shall have the maximum rigidity.

LICK OBSERVATORY,
MT. HAMILTON, CALIF.

ON THE NATURAL CHARGES OF METALS.[1]

BY FERNANDO SANFORD.

THE paper undertakes to show that the phenomena of contact electrification may best be explained on the assumption that different metals (and presumably all other bodies) have definite natural charges of electricity which are capable to inducing charges in other metals when brought near them. The Volta contact charges discovered by Bennett and Cavallo are apparently the bound charges upon the metals due to their mutual induction when they are brought together and their free charges are allowed to escape. The induced free charges were first investigated by Exner and have been discussed by Majorana under the name of approach charges.

Experiments are described by which it is shown that metals while in contact with the earth or with the inside of an earthed hollow conductor may still induce free charges upon other insulated metals near them.

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A., S. August 2–7, 1915.

It is shown that when two metallic bodies are placed within a hollow conductor of another metal their reciprocal induction is modified by the bound charges induced upon both by the surrounding hollow conductor.

It is also shown that a change in temperature of a metal changes the magnitude of its induction upon another metal, and accordingly changes the contact charge between it and the other metal.

It had previously been shown by the writer that the contact electromotive series is the reciprocal of the cohesion series in metals. It is here shown that iron or steel becomes more electronegative with an increase of temperature up to 150 or 200 degrees Centigrade, after which it becomes more electropositive with a rise of temperature. Quotations are given from the writings of other investigators to show that the tensile strength of iron and steel likewise increases with the temperature to about this same point, and that the tensile strength begins to diminish at about the temperature where the temperature coefficient of contact electromotive force changes its sign.

It is suggested that this coincident change of contact electromotive force and cohesion with change of temperature is due to a change in the specific inductive capacity of the metal, and that the Thomson effect in metals may be explained by this change.

LELAND STANFORD JUNIOR UNIVERSITY.

NEW UNITS IN AEROLOGY.[1]

BY ALEXANDER MCADIE.

ANEMOMETRY is in an unsatisfactory state. There is need of instruments giving a record of the *vertical* flow of the air as well as the *horizontal*; and some indication of *gustiness* or discontinuity of flow. The writer criticizes the wind data of all official weather services as incomplete. In general only horizontal change of flow of air is given and then only for eight points of the compass, one direction covering 45°, entirely too large an angle. Directions should be recorded in degrees beginning with the North and progressing through the West to South, this because the prevailing winds are west and the circulation is contra-clockwise or characteristic of storm circulation in the northern hemisphere.

It will be a great step forward when velocities of flow of air or winds are expressed in absolute force units. To accomplish this, the use of pressure tube instruments is advocated in place of revolving cups, it being also pointed out that the correction factor for the latter is in error more than 20 per cent. and therefore official wind data are practically worthless, although much time is spent in accumulating such data.

The writer strongly urges the use at present of the *meter per second* as a velocity unit and the speedy abandonment of miles per hour or feet per second. Nearly all the nations of the world, the United States excepted, use the m./s.

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

Aerostatic pressure, also the vapor pressure, should be expressed in absolute force units, the practical unit being the *kilobar* or 1,000 dynes. The bar thus defined is a strictly C.G.S. unit being the force whlch would impart an acceleration of one centimeter to a gram per second per second. All barometer scales should be regraduated, fractional parts of the inch to be replaced by barokils. Reduction to standard plane of 1,000 kilobars (an absolute atmosphere) can then be accomplished by one operation using some such device as that of Gold of the British Meteor. Office. Fahrenheit and Centigrade temperature scales to be abandoned; and the Absolute Centigrade used, the first of the three figures (generally **2**) to be written once in tabular work at head of column. The use of minus signs for low temperatures (such as frequently occur in winter, also in all upper air work) is thus avoided. It is conceded that the scale division, the Centigrade degree, is too large for many purposes. Possibly later, some value 2.73 times as large as this may be agreed upon and this unit divided into tenths.

The water of the atmosphere in solid or liquid form should be expressed not in fractions of an inch but in millimeters. The United States is the only nation of importance that still records rainfall in inches. Eventually depth of water must be expressed in force units.

BLUE HILL OBSERVATORY,
HARVARD UNIVERSITY.

RADIATION AND ATMOSPHERE.[1]

BY C. G. ABBOT.

A SUMMARY was given of the income and outgo of radiation to and from the earth, its distribution in the spectrum and the effects produced by the various constituents of the atmosphere. Solar radiation ranges principally from 0.3 μ to 3.0 μ, with a spectrum distribution comparable to but differing from that of the perfect radiator at 6500° Abs. C. Terrestrial radiation ranges principally from 5.0 μ to 50. μ, with a spectrum distribution which so far as its source is water (69 per cent.) or vegetation (20 per cent. (?)) is approximately that of the perfect radiator. Naked soil and rock differs from perfect radiator greatly as shown by Coblentz. Mean temperatures on earth's surface range from about 270° to 300° Abs. C. for the various zones of ocean and vegetation. Rock and soil temperatures may run 50° C. higher by day and somewhat lower by night than water and vegetation temperatures. The average spectrum distribution for terrestrial surface radiation is comparable to that of the perfect radiator at 290° Abs. C. Spectrobolometric work, confirmed by self-recording pyrheliometers raised to 25 kilometers altitude by sounding balloons, fix the mean income of radiation from the sum at 1.93 calories per cm.^{2} per min. The solar radiation received varies 7 per cent. per year with solar distance, increases perhaps $3\frac{1}{2}$ per cent. per 100

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., Aug. 2–7, 1915.

sun-spot numbers, and fluctuates from day to day irregularly, sometimes through a range of 10 per cent. in 10 days, but usually much less. Solar variation is confirmed by experiments on relative brightness of centre and edges of sun.

Principal hindrances to direct income of solar rays through the atmosphere are: (1) Clouds; (2) molecular scattering; (3) dust associating itself with water vapor to form haze, or remaining suspended at great elevations after severe volcanic eruptions; (4) selective absorption of water wapor; (5) selective absorption of oxygen; (6) selective absorption of carbon dioxide; (7) selective absorption of ozone. The last three are relatively unimportant. Above 1,700 meters on clear days the losses are almost wholly due to molecular scattering and absorption by water vapor. Fowle computes from observed sky transparency above Mt. Wilson the number of molecules per cm.^{3} at standard conditions as $(2.70 \pm 0.02) \times 10^{19}$. Coefficients of atmospheric transmission for different wave-lengths and figures on the absorption of water vapor at different levels were given.

Principal hindrances to direct outgo of terrestrial rays are: (1) Clouds; (2) water vapor; (3) carbon dioxide; (4) ozone; (5) dust; (6) molecular scattering. Owing to the great wave-lengths concerned, the last two are relatively unimportant. Much investigation is yet required for a satisfactory knowledge of the others. Reference and abstract was given on nocturnal radiation experiments of A. K. Ångström, whose paper is just being published by the Smithsonian Institution.

SOLAR OBSERVATORY OF THE SMITHSONIAN INSTITUTION,
PASADENA, CAL.

SOLAR RADIATION AND TERRESTRIAL MAGNETISM.[1]

BY L. A. BAUER.

DOUBTLESS the chief instruments of research from which we may expect definite knowledge respecting certain salient physical characteristics of the highest levels of the atmosphere, are the self-recording instruments installed at magnetic observatories for the purpose of registering the countless fluctuations to which the earth's sensitive magnetic field is continually subject. The paper shows the rôle which certain terrestrial-magnetic phenomena may play in opening up to us the possible electrical conditions which must obtain in atmospheric regions, 100 kilometers and more above the surface.

Special attention is paid to the conclusions derived from refined magnetic observations made during periods of increased or decreased solar radiation, as, for example, during times of total solar eclipses, or from a concomitant study of variations in the solar constant, as shown by Abbot's observations, and of magnetic fluctuations. It would appear that the Earth's magnetic field undergoes appreciable changes during the time of a total solar eclipse, or

[1] Presented at the joint meeting on Atmospheric Physics of Section B, A. A. A. S. and of the American Physical Society, San Francisco, Aug. 1915.

during the periods of change in solar radiation shown by Abbot's solar-constant values. As a preliminary conclusion it is found that a ten per cent. increase in the solar constant corresponds to a decrease of about one-fiftieth of a per cent. in the magnetic constant employed as a measure of the variations in the Earth's magnetism.

The bearing of a certain class of magnetic disturbances, usually recorded only at observatories in the daylight zone, on the question of the magnitude and duration of variations in the solar radiation, is discussed. In conclusion, it is indicated how magnetic instruments may effectively supplement other appliances in the detection and measurement of solar radiations of various kinds.

The paper will be published in full in Vol. XX (December 1915 issue) of Terrestrial Magnetism and Atmospheric Electricity.

DEPT. TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION, WASHINGTON, D. C.

AN ADAPTATION OF THE KOCH REGISTERING MICROPHOTOMETER TO THE MEASUREMENT OF THE SHARPNESS OF PHOTOGRAPHIC IMAGES.[1]

BY ORIN TUGMAN.

(Communication No. 27 from the Research Laboratory of the Eastman Kodak Company.)

THE Koch registering microphotometer was originally designed to measure the densities of the developed photographic images of spectrum lines, but as the apparatus was used by Koch no attention was given to the resolving power of the apparatus. In many cases inattention to this point will lead to incorrect conclusions drawn from the shape of the registered curve.

The apparatus has been adequately described by Koch in the Annalen der Physik, 39, 1912. The developed negative is passed under an illuminated slit over which is an objective of a microscope which carries a second slit in the focal plane of the objective. The light passing through the microscope falls on a photo-electric cell and generates a current which charges the silvered quartz fiber of a string electrometer. The movement of the quartz fiber is registered on a moving photographic plate. A second photo-electric cell illuminated by the source of lighting the microscope serves as an adjustable leak of the charge on the quartz fiber.

The Koch registering microphotometer was used in the research laboratory of the Eastman Kodak Company to measure the sharpness of photographic images in an investiagtion of resolving power of photographic plates. A carefully ground knife edge was placed on a photographic plate and the whole exposed to a beam of parallel light. The developed negative was passed through the Koch photometer to obtain a curve giving the falling off of density from the edge of the image.

[1] Abstract of a paper presented at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

On an ordinary photographic plate the distance at the edge of an image between maximum and minimum density is less than fifty microns and in many cases not over ten microns. It was necessary, therefore, to use a microscope giving sufficient magnification so that the image of the photographic edge would be several times the width of the slit in the focal plane of the microscope objective. It was found that the width of this slit could not be less than .75 mm. when using a four-mm. objective because of the pinhole image formed of the back surface of the objective on the photo-electric cell. If the magnification is less, the width of the slit cannot be less because of the pinhole image formed. With a constant slit width the resolving power of the apparatus is limited by the magnification of the microscope objective.

The registered curve of any density gradient will be in error over a distance (d) at each end represented by $d = s/m$ where (s) is width of slit and (m) is the magnification. If L is the distance over which the density gradient extends the fraction of the curve in error is d/L.

RESEARCH LABORATORY,
EASTMAN KODAK COMPANY,
July 20, 1915.

THE RESOLVING POWER OF PHOTOGRAPHIC PLATES.[1]

BY ORIN TUGMAN.

THE resolving power of photographic plates has hitherto been investigated by methods which have been suggested by general definitions. The reduced images of fine-lined structures have been photographed and examined under a microscope to determine what lines are resolved. This method is accurate only to the limits of personal judgment and does not reveal the properties which determine the resolving power of an emulsion.

A departure from this method was made by Goldberg[2] who made contact prints through small conical holes in a metal plate. The increase in diameter of the image as the time of exposure was increased was measured and a curve plotted with increase of decimeter against exposure. The slope of this curve was called the turbidity factor and was defined mathematically as $dx/(d(\log I))$ where x is distance from the edge of the image and I is intensity of light measured laterally from the exposed area. The sharpness factor dD/dx is put equal to the photographic factor $dD/(d(\log I))$ divided by $dx/(d(\log I))$, where D is density of the negative. It can be shown that $dx/(d(\log I))$ is not the slope of the curves drawn by Goldberg.

Nutting[3] pointed out that by simple differentiation the density gradient is

$$\frac{dD}{dx} = \frac{dD}{d(\log E)} \frac{d(\log E)}{dx}$$

[1] Abstract of a paper presented at the meeting of the American Physical Society at San Francisco, August 2–7, 1915.

[2] Goldberg, Photo. Jour., 36, 1912.

[3] Nutting, Photo. Jour., June, 1914.

where E is exposure and that the investigation of resolving power involved a study of the density gradient.

It was argued that inasmuch as the photographic gradient $dD/(d(\log E))$ increases with development and that $(d(\log E))/dx$ is not a function of development the density gradient should increase with development.

A carefully sharpened steel edge was laid on a plate and exposed to a parallel beam of monochromatic light. The density gradient at the edge of the image was measured on the Koch[1] registering microphotometer and also, a Königs-Martens photometer modified for microphotometry. Under ordinary exposures the plates showed no increase of density gradient with development. The density gradient obtained with violet light was greater than with green light in all the cases tried. A process plate dyed yellow gave the greatest density gradient of all plates tried. This may be explained by the increase of optical opacity of the emulsion. The light scattered by the silver halide grains is not affected by the yellow dye but the diffused light is absorbed by the yellow dye and as a result there is very little spreading of the image.

RESEARCH LABORATORY, EASTMAN KODAK CO.,
July 22, 1915.

ON THE ORIGIN AND MAINTENANCE OF THE EARTH'S NEGATIVE CHARGE.[2]

BY W. F. G. SWANN.

The first portion of the paper is devoted to an examination of the possible sources in which an explanation of the observed phenomena of atmospheric electricity may be sought, and to a general discussion of certain broad principles which largely restrict the type of theory available. A brief discussion of former theories is also given in the above light. The second portion of the paper comprises a more detailed consideration of a certain class of hypotheses provisionally formulated for the coördination of atmospheric-electric phenomena. The paper will be published in full in Vols. XX and XXI of Terrestrial Magnetism and Atmospheric Electricity.

DEP'T TERRESTRIAL MAGNETISM,
CARNEGIE INSTITUTION,
WASHINGTON, D. C.

REVIEW OF LABORATORY STUDIES ON THE ZEEMAN EFFECT AT MOUNT WILSON SOLAR OBSERVATORY.[2]

BY HAROLD D. BABCOCK.

A BRIEF account of apparatus and methods used in work on the Zeeman Effect, including descriptions of spectrograph, magnet, means for production of sparks, analyzing devices and other auxiliaries, was followed by illustrations of selected portions of the spectra of iron, vanadium and chromium

[1] Koch—Annalen der Physik, 39, 1913.

[2] Presented at the meeting on Atmospheric Physics of Section B, A. A. A. S., San Francisco, 1915.

as affected by a magnetic field. Special attention was directed to the method of standardizing the magnetic fields, which involved the use of a ballistic galvanometer, a number of test coils and a standard mutual inductance in whose primary circuit known currents of electricity could be reversed at will. A small correction was applied to allow for the variation of the field intensity over the volume occupied by the exploring coil, the determination of its amount being made by means of differential measurements with a bismuth spiral. Effects due to variation of field strength over the space occupied by the radiating vapor in the spark were shown for the case of the strong violet lines of chromium, which are very easily reversed.

Numerous lantern slides were used to illustrate the effects discussed. Typical lines ranging from the class unaffected by a magnetic field to the line of twenty-one components were shown, as well as examples of the use of various analyzers over the slit. Methods of using the echelon and the Fabry-Perot interferometer in the study of weak magnetic fields were also described. In view of the unfinished state of the work, definitive data were not presented.

MT. WILSON SOLAR OBSERVATORY,
PASADENA, CAL.

NOTE ON THE THEORY OF THE CORONA.[1]

BY W. P. ROOP.

THEORIES of the corona have been advanced by Townsend[2] and by Bergen Davis.[3] By strengthening the theoretical basis of one of Townsend's assumptions, and by calling attention to certain features of Davis's treatment, I believe that I can definitely establish the correctness of Townsend's theory.

Townsend's theory is based on the usual empirical equation embodying the results of Baille and Liebig on variation of sparking gradient between parallel plates with distance between the plates. This equation may be given a theoretical basis in the following way.

The current passing between parallel plates is proportional to the expression[4]

$$n = n_0 \frac{(\alpha - \beta)e^{(\alpha-\beta)d}}{\alpha - \beta e^{(\alpha-\beta)d}},$$

where n_0 is the number of ions supplied per second at the negative plate, d is the distance between the plates, and α and β are the numbers of ionizing impacts per centimeter for negative and positive ions respectively. α and β are functions of the gradient X which may be expressed as follows:[4]

$$\alpha = Ne^{-NV/X}$$

and

$$\beta = Me^{-MU/X}.$$

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

[2] Townsend, Electrician, June 6, 1913.

[3] Davis, Proc. A. I. E. E., March, 1914.

[4] Townsend, Theory of Ionization by Collision.

Here N and M are the reciprocal free paths, and V and U are the potential falls required to produce ionization, for the negative and positive ions respectively.

If d is held constant, and X gradually increased, a stage is reached at which the denominator of our expression for current becomes zero, and the spark passes. If we now denote by α and β the limiting values at the sparking gradient, we have the relation

$$d = \frac{1}{\alpha} ln \frac{\alpha}{\beta}$$

which becomes, on substitution,

$$d = \frac{e^{NV/X}}{NX}\left(Xln\frac{N}{M} - NV + MU \right),$$

or, approximately,

$$d = \frac{e^{NV/X}}{X}\left(\frac{M}{N}U - V \right).$$

The following table exhibits (1) observed sparking distances for certain values of gradient,[1] (2) the corresponding calculated values, by the empirical formula, and (3) the values calculated by the theoretical formula.

Obs.	Calc. Emp.	Calc. Theoret.
.044	.052	.027
.084	.082	.069
.100	.100	.100
.186	.150	.201
.240	.180	.261
.325	.220	.338
.471	.295	.463
.559	.357	.555
.623	.403	.612
.741	.455	.668
.883	.610	.805
1.067	1.29	1.089
1.144	18.5	1.179

The empirical formula is thus seen to be approximately justified, though the theoretical formula gives closer agreement with observed data.

The calculated results are based on constants determined so as to make the calculated and observed data agree at two points. From the values of the constants thus determined, V, the potential fall required to impart ionizing energy to a negative ion, is calculated to be 7.8 volts. This constitutes an independent determination of this quantity. It is based on the assumption that the negative ion has a mean free path four times as great as the accepted value for a neutral molecule.

The essential difference between the two theories is that according to Davis,

[1] Liebig, Phil. Mag., 24, 111, 1887.

ionization by negative ions only occurs, whereas according to Townsend, ionization by positive ions as well is necessary. It is now clear that as far as the discharge between plates is concerned, the discharge sets in when β reaches a certain small value, which nevertheless differs appreciably from zero. It is by the assumption that the conditions about a wire discharging by corona approximate those between plates that Townsend reaches theoretical conclusions agreeing with the observations of Peek and Whitehead.

The apparent success of Davis's theory seems to be attributable to the fact that the condition arbitrarily assumed to exist when corona appears agrees in form with the condition here deduced. Davis assumes corona to set in when the number of ions produced by a single ion in reaching the wire from a distance attains a certain constant value. In point of fact, it is not this, but the ratio of α to β which must reach a given value. Since Davis's theory is based on the calculation of this constant, the accident by which it does not receive its true meaning happens not to interfere with the success of the calculations in reproducing observed data.

University of California,
Berkeley, Cal.,
August 14, 1915.

The Resistance of a Spark Gap.[1]

By W. P. Boynton.

THE behavior of an oscillatory electric circuit is usually discussed on the assumption that the rate of dissipation of energy from the circuit is such as would result from a constant value of the resistance of the circuit, with no losses due to other causes. Many determinations have been made of the equivalent resistances of spark gaps in oscillatory circuits, which have established the fact that these resistances may be and usually are comparatively small. Such values are however not a satisfactory description of such spark resistances, which in all probability have very low initial values, suddenly attained, and then increase rapidly to an infinite value.

As a second approximation to the behavior of such sparks the writer[2] has suggested the assumption that the resistance is constant for a limited period, suddenly becoming infinite. While the number of oscillations to be expected on such an assumption seemed to agree fairly well with the spark photographs available for comparison, this was not satisfying as a description.

The assumption of a constant resistance leads to a graph of the oscillations in which the envelope is the familiar exponential curve, but Zenneck[3] has found experimentally that "when an oscillatory circuit contains a spark gap the decadence of the maxima is approximately represented by a straight line."

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

[2] Phys. Rev., 1st Ser., XIX., p. 191, 1904.

[3] Ann. der Physik, 13, p. 822, 1904.

The purpose of this paper is to investigate the values of spark resistance consistent with this observation.

Supposing the circuit to consist of a condenser with its two coatings connected through a metallic inductive circuit and a spark gap, the assumptions that the current may be represented by

$$I = {}_0(l1 - at) \sin bt)$$

and the difference of potential at the terminals of the condenser by

$$V = V_0 \left[(1 - at) \cos bt + \frac{a}{b} \sin bt \right]$$

are consistent with each other, if

$$I_0 = - b C V_0$$

and give satisfactory initial conditions, besides satisfying Zenneck's description, and when substituted in the differential equation

$$V + RI + LdI/dt = 0$$

lead to the results that

$$b = 1/LC \quad \text{and} \quad R/2L = a/(1 - at)$$

and hence that initially, when $t = 0$, $R_0/2L = a$, and hence

$$R = R_0/(1 - at)$$

that is, the resistance of the spark has a finite initial value R_0, and increases regularly, becoming infinite at the time $t = 1/a$, its graph being an equilateral hyperbola, while its conductance decreases uniformly in the same time from its initial value $1/R_0$ to zero. This description of the spark seems plausible, and consistent with our notions of its mechanism, as made up of electrons set free in large numbers at the first passage of the spark, and gradually dissipated, either spontaneously or by whatever means was taken to prevent the persistence of the arc, when that is necessary.

The effective current as shown by a hot-wire ammeter due to a succession of such oscillation trains occurring regularly N times per second would be given by the formula

$$I^2 = NI_0^2/6a = C V_0^2 N/3R,$$

from which the initial resistance is

$$R_0 = NC V_0^2/3I^2,$$

in which all the quantities on the right may be determined experimentally.

One of the interesting applications of this study of spark behavior is to the theory of the optimum conditions for producing long wave trains in a radiotelegraphic transmitting system inductively excited. It is well known that for the practical attainment of this end two adjustments are required, first the tuning of the two circuits to resonance, and second the adjustment of the coupling or mutually inductive effect until a maximum effect is produced in the secondary circuit, as shown usually by a hot-wire ammeter.

It has been shown[1] in the case of two undamped oscillating systems of the same free period with sufficiently weak coupling that there is a surging of energy alternately from one system to the other, such that corresponding states of the two systems may be represented by the forms

$$A_1 \cos mt \cos bt \quad \text{and} \quad A_2 \sin mt \sin bt,$$

in which in our problem $b = \sqrt{1/L_1C_1} = \sqrt{1/L_2C_2}$ and $m = bM/2\sqrt{L_1L_2}$, and $A_2/A_1 = \sqrt{L_2/L_1}$. This may be interpreted of course as two oscillations of the same period but differing in phase by $\pi/2$, and with amplitudes which also vary harmonically with the same phase difference. In the practical case the oscillations are damped, and the primary spark quenched, so that the primary circuit is losing energy both by dissipation and by transfer to the secondary, and its oscillations cease completely when its spark is quenched. The loss by dissipation would correspond to an envelope containing the factor $(1 - at)$ while that due to a transfer to a secondary which was itself radiating and dissipating energy would suggest a factor $e^{-ct} \cos mt$, where m has the same significance as above, and c depends upon the rate of loss of energy in the secondary circuit. It will appear that these two envelopes may be of almost the same shape, and hence we may give the following description of the behavior of the two circuits, which while not exact is yet a closer approximation than is usually given:

$$V_1 = V_{01}(1 - at) \cos bt,$$

$$I_1 = -bC_1V_{01}(1 - at) \sin bt,$$

$$V_2 = V_{02}e^{-ct} \sin mt \cos bt,$$

$$I_2 = -bC_2V_{02}e^{-ct} \sin mt \sin bt,$$

in which the terms omitted have contained factors of the order of a/b, c/b, m/b. The optimum effect depends upon such an adjustment of the coupling that $m = \pi a/2$, so that at the time $t = 1/a$ the factor $(1 - at)$ will vanish and sin mt will become unity. To the first condition corresponds the opening of the primary circuit by the quenching of its spark, so that it no longer receives energy from the secondary, whose further behavior is represented by

$$V_2 = V_{02}e^{-ct} \cos bt,$$

$$I_2 = -bC_2V_{02}e^{-ct} \sin bt.$$

This system of six equations then very closely represents the decrease of the primary oscillation due to dissipation and transfer of energy, the increase of spark resistance to its infinite value, the increase of the secondary oscillation due to transfer of energy from the primary, and its gradual decay due to dissipation and radiation.

While the curve representing the function $e^{-ct} \cos mt$ when $m = \pi a/2$ lies fairly close to the straight line $1 - at$ for a considerable range of values of c, especial interest seems to attach to the case in which $c/a = (\pi/2)e^{-c/a} = .7454$.

[1] Rayleigh, Theory of Sound, I., p. 164.

For this value the curve cuts the straight line at $t = 0, 4/9a, 1/a$, and its largest variation is at $t = 7/9a$, the curve giving .191, the line .222, the variation being about 20 per cent. of the odinate, but only 4 per cent. of the largest value.

While for undamped oscillations the ratio of transformation would be $V_{02}/V_{01} = \sqrt{L_2/L_1}$, the maximum secondary E.M.F. would be attained at the time $t = 1/a$ when e^{-ct} becomes $e^{-c/a}$, whose value for $c/a = .7454$ is .4746, hence a probable value would be

$$V_{2max} = .4746\ V_{01}\sqrt{L_2/L_1}$$

or the ratio of transformation would be very nearly one half that for undamped oscillations.

UNIVERSITY OF OREGON,
EUGENE, OREGON,
June 22, 1915.

ELECTROMOTIVE FORCES IN ISOTHERMAL METALLIC CIRCUITS.[1]

BY GILBERT N. LEWIS.

THE accepted statement that electromotive forces can occur in completely metallic circuits only when different parts of the circuit are at different temperatures, is true only when no process involving a diminution in free energy accompanies the transfer of electricity. This is not the case when two amalgams of different concentration are adjacent portions of the electrical circuit. Experiments made in this laboratory by Dr. Adams and Miss Lanman show that a measurable transfer of the dissolved metal accompanies the passage of the current through an amalgam. From such measurements can be calculated the potential of a concentration cell composed of two amalgams of the same metal in contact and connected by platinum wires. The potential thus calculated proves to be an easily measurable quantity, and its experimental determination for a number of amalgams will, it is hoped, afford a means of finding the concentration of electrons in pure mercury.

LABORATORY OF CHEMISTRY,
UNIVERSITY OF CALIFORNIA.

THE SPECTRUM OF THE "TUBE-ARC," AND A COMPARISON WITH LINE-DISSYMMETRIES IN SPARK SPECTRA.[1]

BY ARTHUR S. KING.

THE "tube-arc" is formed when a graphite tube enclosed in a vacuum chamber is burned apart at its middle, with the formation of an arc carrying a large current at low voltage. Substances placed in the tube at the point of rupture are thus vaporized and many peculiarities in their spectra are observed.

The chief features of this source have been previously reported. In a recent

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

study of the spectrum of iron in the tube-arc, special attention was paid to the dissymmetry produced in many of the stronger lines. The density-curves given by the micro-photometer for these lines show their dissymmetry to be equivalent to a displacement of the maximum, usually toward the red. The effect differs greatly in amount for various lines and the grouping on this basis is found to be closely related to the temperature classification of the lines in the furnace spectrum, and less directly to the amounts of their displacement under pressure. The effects resemble closely those observed at the pole of the iron arc, both as to dissymmetry of lines and their relative intensities.

It was found for iron and other elements that a powerful and disruptive spark produces line dissymmetries which are very similar to those of the tube-arc. The appearance of the characteristic spark lines in the tube-arc is a further resemblance to the spark. As the tube-arc is a low-potential source, the similarity to the spark effect must be looked for in a cause other than high potential gradient. The effects of high electronic speed are being investigated, as this may be a common feature of the two sources.

MOUNT WILSON SOLAR OBSERVATORY.

A SUMMARY OF THE LEADING FEATURES OF ELECTRIC FURNACE SPECTRA.[1]

BY ARTHUR S. KING.

THE vaporization of metals in the tube resistance-furnace has been found to be a very effective method of studying the spectrum under conditions which differ in many respects from those prevailing in the flame, arc, or spark. The temperature of the vapor in the carbon tube may be controlled through a wide range, and the resulting changes in the spectrum have been observed for a number of elements. It is found that, beginning with a few lines at low temperature, the richness of the spectrum may be gradually increased until the number of lines is comparable with that given by the arc, but many differences in relative intensity of lines are found between the furnace and arc spectra.

The features given special study in the spectra at various furnace temperatures are the successive appearance of groups of lines as the temperature rises, and the varying rates of increase in the intensity of lines belonging to different groups. The classification arrived at in this way is found to be closely connected with the behavior of these groups of lines under other physical conditions.

In addition to the variation of spectra with temperature, the furnace has been found well adapted to the study of absorption effects, pressure displacements of either emission or absorption lines, the effect of various atmospheres, banded spectra, and the spectra of mixed vapors.

MOUNT WILSON SOLAR OBSERVATORY.

[1] Abstract of a paper presented at the San Francisco joint meeting of the Physical Society and Section B of the A. A. A. S., August 2–7, 1915.

THE THUNDERSTORM.

BY W. J. HUMPHREYS.[1]

ACCORDING to Simpson's well-supported theory, the electric separation that gives rise to lightning is produced by the disruption of raindrops in the air and the transfer of the resulting negative spray to higher altitudes by convection currents. From this theory it follows, in full accord with observation, that thunderstorms must be confined almost exclusively to large cumulus clouds, and therefore occur most frequently when the surface temperature and the absolute humidity both are high.

Such peculiar phenomena of the thunderstorm as the gentle breeze toward it, the more or less violent wind gusts immediately in front of and away from it, the abrupt drop in temperature, the equally abrupt rise in pressure, the occurrence of hail and the various forms and movements of the accompanying clouds may all be explained as consequences of a rising current starting in front of the storm and becoming especially vigorous within the cumuli, together with a corresponding down rush of totally different air from within and below the lower cloud levels. The temperature contrast essential to this peculiar circulation is maintained by the simultaneous action of a heat source, condensation, and a heat sink, evaporation; the former occurring within the cloud; the latter all the way between the cloud and the surface.

Lightning, in its various forms, though merely incidental to and in no sense the cause of the wind and rain of a thunderstorm, often is the most impressive of all its phenomena. When analyzed by a rotating camera its vivid sinuous streaks are either single, as they appear to the eye; multiple, consisting of several single streaks occurring along identical paths but at unequal intervals; or complex, producing a picture analogous to that of a waving, plaid ribbon. Occasionally a streak of lightning persists during an appreciable length of time and then gradually fades away, thus producing through its end-on and therefore brighter portions, the phenomenon known as beaded or pearl lightning.

Sheet lightning, as seen in distant clouds, appears to be only the diffused and reflected light of ordinary streak lightning. Rocket, or slow moving, lightning and ball lightning have too often been reported by good observers to permit of their classification with things wholly subjective, however baffling at present may be the difficulty of explaining them.

It is often stated that the lightning discharge consists of many high frequency oscillations. There is much evidence, however, in favor of the idea that the discharge is either unidirectional or else very heavily damped. The question is still an open one, as are many others in connection with the thunderstorm, and should tempt further and conclusive investigation.

[1] Abstract of a paper presented at the joint meeting of the Physical Society and Section B of the A. A. A. S., in Berkeley, August 3, 1915.

NEW BOOKS.

Prinzipien der Atomdynamik. III Teil. By J. STARK. Leipzig: S. Hirzel, 1915. Pp. xvi+280. Price, 9 Mk.

This volume, which has to do with "Die Elektrizität in Chemischen Atom" and is illustrated with ninety-four diagrams constitutes an attempt on the part of the author to explain a number of the properties of matter from a physical, chemical, and crystallographical standpoint by means of certain assumptions regarding the manner in which valency or surface electrons are bound to atoms and to the role these play in molecular groupings of atoms. Among the subjects treated are the arrangement and energy of the valency electrons in the benzol ring, electrolytic dissociation in solutions, catalysis, conduction of electricity in metals, the relation of band spectra to the chemical constitution of molecules, optical dispersion, molecular and atomic refraction, and the rotation of the plane of polarization of light by different substances. The assumptions made by the author in regard to the character and functions of the electronic bonds are remarkably simple, and with these he has succeeded in linking up a mass of scattered and hitherto disjointed information regarding a great number of very diverse phenomena. J. C. McL.

Physical Laboratory Manual. CHAFFEE. Cambridge: Harvard University Press, 1914. Pp. vi + 128.

Harvard led the way among American Universities in making a sharp distinction between beginning, or first-year physics, offered in that institution for pupils who do not present physics for entrance, and the second-year course assigned for those who have had the beginning course. The present manual contains the directions for the 34 laboratory experiments in this second-year course. It is apparently the successor to Professor Sabine's manual and represents a distinct advance in the choice of material for such a course and in the elimination of material which is on the whole better adapted to the beginning course. It is on this choice that the chief merits and demerits of college manuals lie. The judgment shown in Professor Chaffee's manual is especially commendable. R. A. M.

X-Rays and Crystal Structure. W. H. BRAGG AND W. L. BRAGG. London: G. Bell and Sons, Ltd., 1915. Pp. vii + 228.

Of the numerous investigators who have applied Laue's brilliant discovery to the investigation of X-rays and of crystalline structure, none have been so successful in disentangling the many complications of the subject and in advancing rapidly and surely toward a completer knowledge as the two Braggs,

father and son. It is well known that their work was greatly facilitated by the fortunate "reflection" point of view due to W. L. Bragg. It is very interesting to observe that this is mathematically identical with the theory first given by Laue; the only change is a transformation of axes such that, along two of the axes, the interference of the ray considered shall be of zero order. But, as not infrequently happens, this apparently trivial change makes all the difference between fertility and rapid progress on the one hand, and confusion and delay on the other.

No one who reads the present book, however, will remain under the delusion that the success of the authors in their investigations is due entirely to a fortunate point of view; on the contrary, one is constantly moved to admiration by the acuteness and insight which they display and the directness with which they proceed to their goal, both in experimental methods and in the interpretation of their results.

After a brief introductory chapter, a simple theory of the diffraction of the rays by the crystal atoms is given and the construction and use of the X-ray spectrometer is described. A brief summary of the properties of X-rays—especially the work of Barkla—is contained in Chapter IV. The following chapter is devoted to an elementary but very clear exposition of crystal structure and of the notation used by mineralogists; although it deals only with very simple forms, this chapter will doubtless be useful to many physicists whose knowledge of crystallography, like that of the present reviewer, is rudimentary. Chapter VI deals with X-ray spectra and the interesting relations which have been found to subsist among the characteristic radiations of different elements. The next four chapters take up in detail the question of the analysis of crystal structure; the ingenuity and resourcefulness shown in these pages is very remarkable. In Chapter XI the authors discuss the intensity of X-ray reflection and its dependence on various factors. The book closes with an analysis of some of Laue's photographs from the "reflection" point of view. H. A. B.

An Introduction to Laboratory Physics. LUCIUS TUTTLE, Associate in Physics, Jefferson Medical College. Philadelphia: Jefferson Laboratory of Physics. Pp. xi + 150.

A series of fifteen excellent exercises on the scientific treatment of data in physical measurements, a very handy and useful little book for every student of laboratory physics beginning or advanced. R. A. M.

Annuaire Pour L'An 1915, Publie Par Le Bureau Des Longitudes. Avec une Notice scientifique. Paris: Gauthier-Villars, 1915.

The main portion of the volume, namely 754 pages, consists of the usual astronomical tables, weights and measures, interest and actuaries tables. Appended thereto is a treatise of 173 pages entitled "Les méthodes d'examen des miroirs et des objectifs" by M. G. Bigourdan, which merits examination by those in this field of work. F. B.

Vocational Mathematics. By WM. H. DOOLEY. Boston: D. C. Heath and Co., 1915. Pp. viii+341.

This book has been prepared to meet the criticism that a pupil graduates from the usual course in mathematics without being able to "commercialize" or apply his knowledge in such a way as to be able to meet the needs of trade and industry. F. B.

Theory of Measurement—A Manual for Physics Students. By JAMES S. STEVENS, Professor of Physics, University of Maine. Pp. vi + 80. D. Van Nostrand Co. $1.25.

A very practical little laboratory book on the Theory of Probability in its relation to the treatment of errors in physical measurements. Not so well adapted to beginners as Tuttle's book but excellent for those having some knowledge of the calculus. R. A. M.

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Vol. V., May, 1915, article by S. A. Moss, entitled "Integration of Thermodynamic Equations for an Imperfect Gas"; the last term in equation 25, page 444, should be divided by $f^{-1}(p)$ so as to read $+ Tv_p / f^{-1}(p)$.

Vol. VI., October, 1915, article by H. O. Taylor, entitled "A Mechanical Process for Constructing Harmonic Analysis Schedules for Waves Having Even and Odd Harmonics":

p. 304, equation (3), second term of right-hand member, should read, $C_2 \sin(2\omega t + \alpha_2)$;

p. 305, schedule for A_9, v_6 should be $+$;

p. 305, value of $y_{3n/2}$, last term should read, $+ B_n$;

p. 308, line 19, should read, at the beginning *of counting*;

p. 309, mark for second footnote should be 2.

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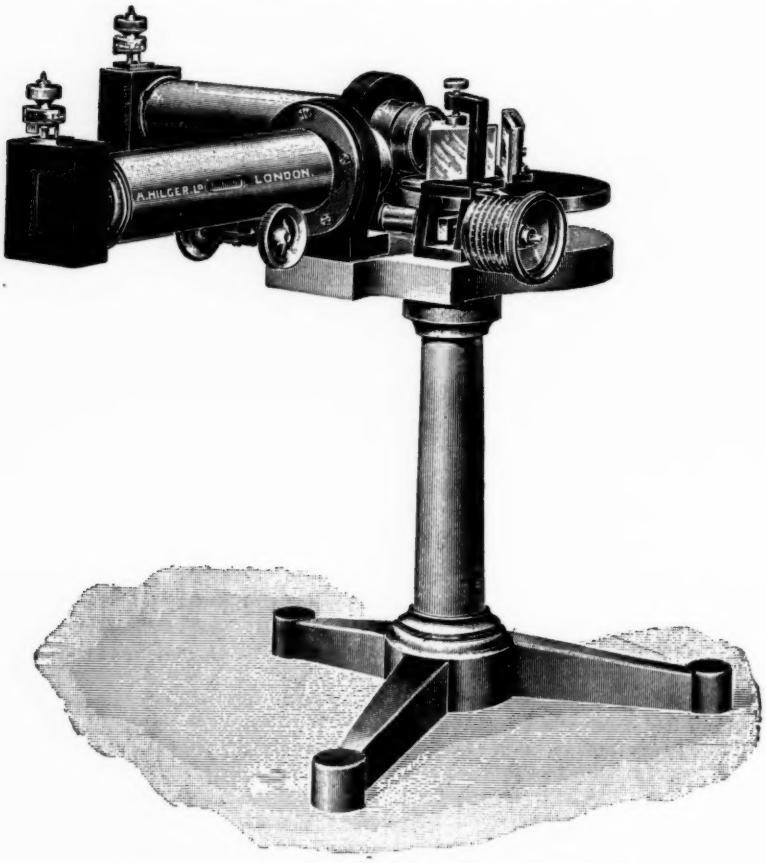

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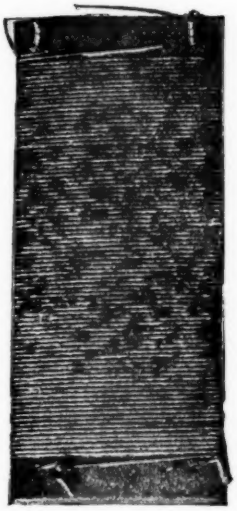

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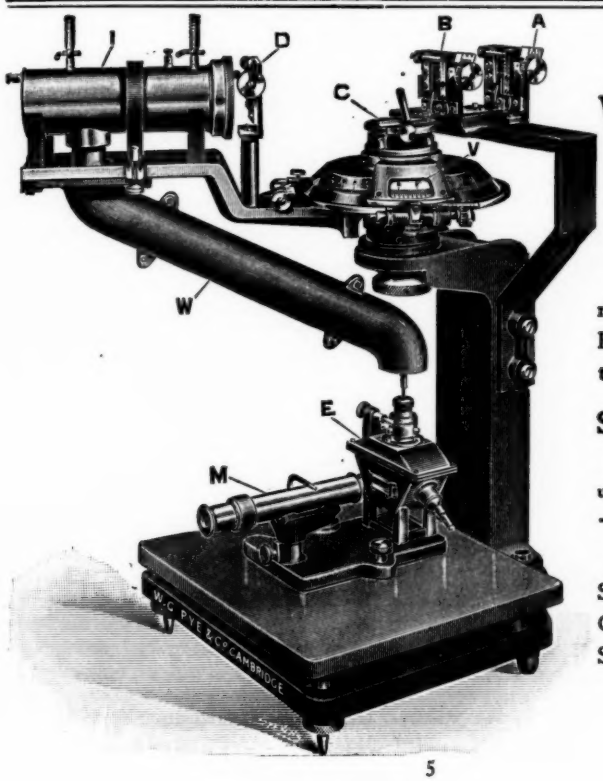

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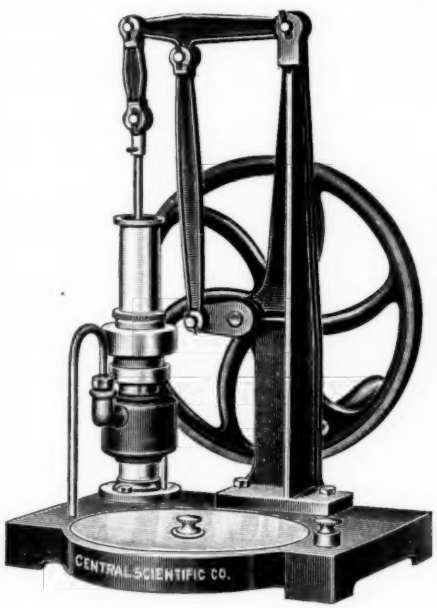

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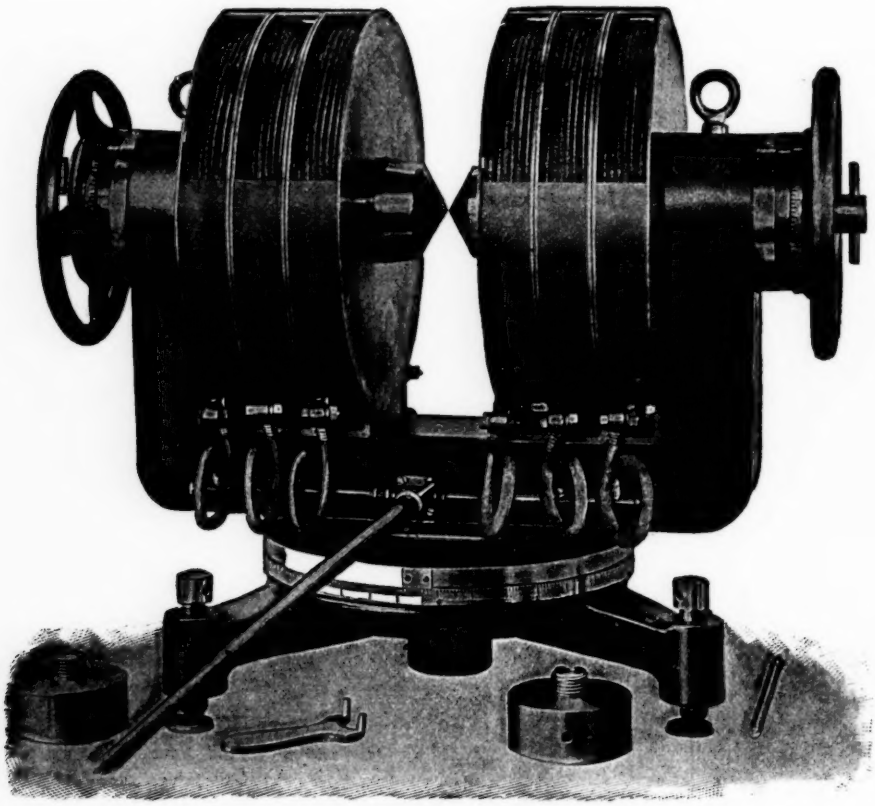


New Design of Prof. P. WEISS Electromagnet

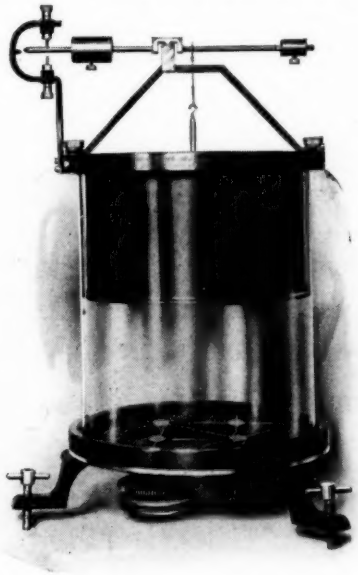
Winding of coils with hollow copper, traversed by the electric current and the cooling water Absolutely no heating of the magnet and constant field whatever may be the length of the experiments. This design is constructed in various sizes up to 145 mm. pole diameter.

The 120 mm. type gives fields as follows:—

Size of the air-gap		
Length.	Diam.	Fields
60 mm.	120 mm.	9530 gauss
10 "	120 "	18875 "
35 "	35 "	16000 "
35 "	10 "	25900 "
35 "	3 "	29630 "

Size of the air-gap		
Length.	Diam.	Fields
15 mm.	10 mm.	26600 gauss
2 "	10 "	41000 "
5 "	3 "	37510 "
1.5 "	3 "	52500

NORTHRUP VISIBLE MOLECULES APPARATUS



Fully illustrates, in a visible form, the motions of gas molecules and the principles which govern these motions. The molecules of a gas are visibly represented by a large number of small steel balls, which are maintained in continual motion within a glass cylinder by the action of four rotors. The distribution of these balls is perfectly uniform. The apparatus may be used to illustrate the change of pressure with constant volume, the change of volume with constant pressure, the viscosity of a gas, and the phenomenon of the Brownonian Move_ments. In addition, a number of experiments may be performed to test the correctness of several corollaries of the kinetic theory.

This is the *new apparatus* which was demonstrated by Dr. E. F. Northrup, of Princeton University, at the New York meeting of American Physical Society on October 30th, 1915.

Full particulars in Bulletin 836, *just published.*

Please write for a copy if interested.

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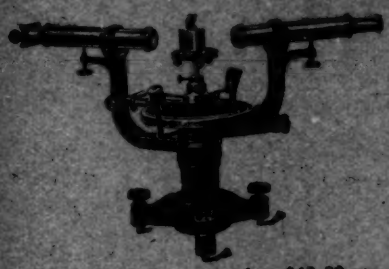

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The movable system has an extremely low moment of inertia and is very effectively damped. Indications are independent of room temperature, the heating effect of current passing through the coils, and the instrument is shielded from external magnetic influences.

The scale which is 5¾ inches long, is uniform throughout the *entire* length, a characteristic of great value. It is hand-calibrated and provided with a mirror over which the knife-edge pointer travels, and the pointer may easily be adjusted to zero by means of a zero-correcting device.

For complete information regarding Model 329 Wattmeters write for Bulletin 2002. Other models in this group are Model 310 Single-Phase and Direct Current Portable Wattmeter, described in Bulletin 2002; Model 370 A. C. and D. C. Ammeter, described in Bulletin 2003; and Model 341 A. C. and D. C. Voltmeter, described in Bulletin 2004.

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